Teaching for Robust Understanding with Lesson Study

Alan H. Schoenfeld
University of California
Berkeley, CA, USA
Alans@Berkeley.Edu

First, Thanks!

My goal for today:

To show that Lesson Study and the TRU framework are a marriage made in heaven.

What really matters in Lesson Study?

Lesson Study is a collegial activity.

Teachers are honored as professionals.

The goal is enhanced student and teacher learning.

More specifically, there is:

- An emphasis on richly conceived and connected mathematics.

What really matters in Lesson Study (continued)?

- A central focus on student thinking, in the service of students developing powerful mathematical understandings.
- Careful attention to the activities students will engage with, to support that learning. (Research lessons, aimed)
- Careful refinement of research lessons, based on evidence of how students interact with the lesson.

That is:

Working on a Lesson Study lesson provides a “deep dive” into student thinking about important mathematics, working on how to “meet students where they are.” It does so as part of a PLC.

Now I'll turn to TRU.

Let's begin with this question:

If you had 5 things to focus on in order to improve students' classroom learning, what would they be?

And,
How would you know
they're the right things, or that you're
not missing anything important?

Why 5 (or fewer)?

It's as many as most folks can keep in mind. (In fact, it may be too many to work on at one time.)

If you have 20, you might as well have none. People can't keep that many things in their heads, and long check lists don't help. What matters is what people can act on, in teaching and coaching.

What properties should those 5 things have?

They're all you need (there's nothing essential missing).

They each have a certain "integrity" and can be worked on in meaningful ways.

Their framing supports professional growth.

You're about to meet the
Teaching for **R**obust **U**nderstanding of **M**athematics
(TRU Math)
framework

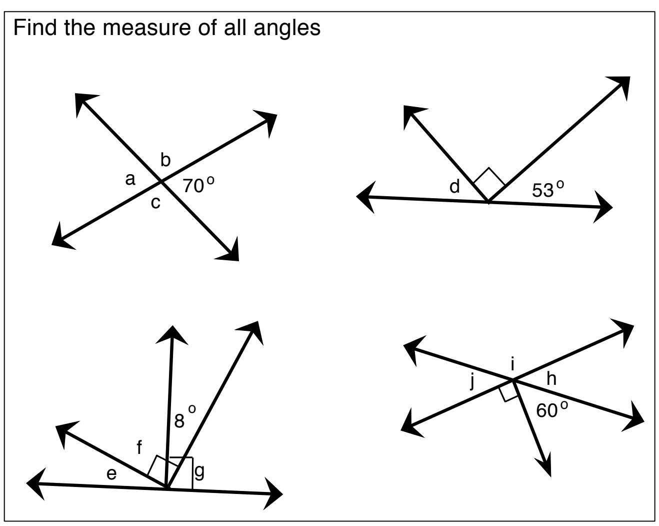
If we had a lot of time, we would look at a bunch of videos and discuss what we see in them.

But we don't. So, I'll show snippets from two lessons you may be familiar with, and focus on a 6th grade classroom you may know.

Tape 1: The TIMSS Geometry Video

This is a typical U.S. lesson about finding the values of complementary, supplementary, and vertical angles.

The TIMSS Geometry Video

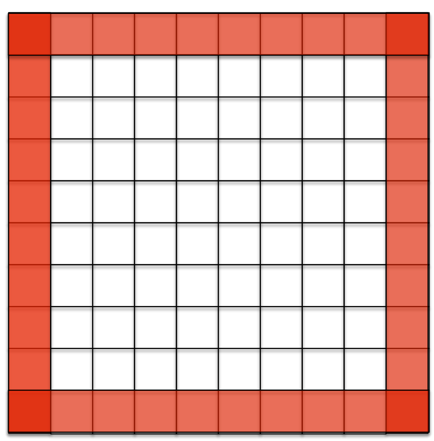


The Geometry Lesson

Tape 2: The Border Problem from *Connecting Mathematical Ideas* by Jo Boaler and Cathy Humphreys

Here's a 10 x 10 grid.

How many border squares are colored in?



The Border Problem Lesson

Tape 3: a 6th grade classroom at Reinberg Elementary School.

The context:

a "Formative Assessment Lesson" entitled "translating between fractions, decimals, and percents."

This lesson is available for free, along with 99 other formative assessment lessons (a.k.a. "Classroom Challenges"). Just google "mathematics assessment" to find the Mathematics Assessment Project website.

To date we have more than 5,000,000 lesson downloads. (More later.)

The task starts with decimals and percents.

0.2 / ___%	0.05 / ___%	_.___ / 80%
0.375 / ___%	_.___ / 12.5%	0.75 / ___%
1.25 / ___%	_.___ / 50%	_.___ / ___%

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Working Together 1

Take turns to:

1. Fill in the missing decimals and percents.
2. Place a number card where you think it goes on the table, from smallest on the left to largest on the right.
3. Explain your thinking.
4. The other members of your group must check and challenge your explanation if they disagree.
5. Continue until you have placed all the cards in order.
6. Check that you all agree about the order. Move any cards you need to, until everyone in the group is happy with the order.

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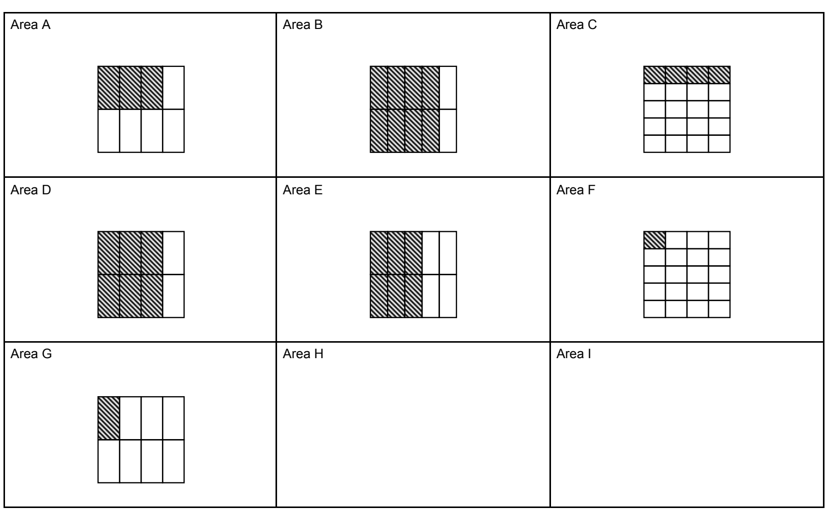
Then students are given area cards,



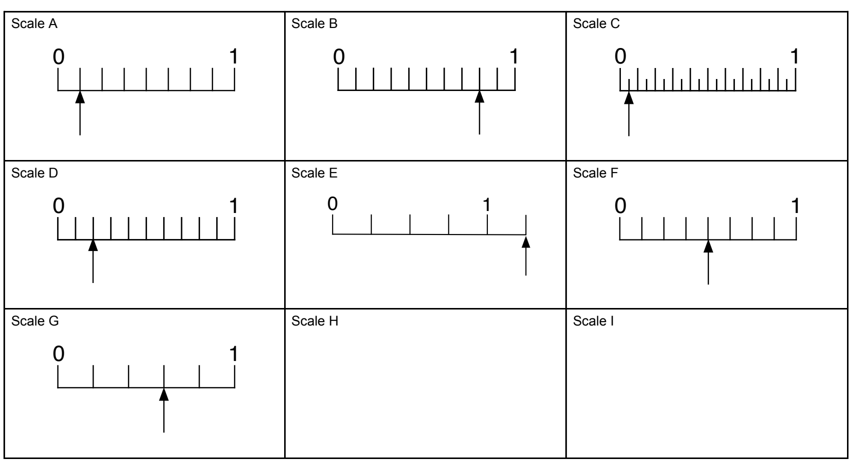
P-24

Fraction cards,

$\frac{3}{8}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{6}{10}$	$\frac{5}{4}$
$\frac{1}{8}$		

P-25

And scales,



And asked to order them all.

P-26

Working Together 2

Take turns to:

1. Match each area card to a decimals/percents card.
2. Create a new card or fill in spaces on cards until all the cards have a match.
3. Explain your thinking to your group. The other members of your group must check and challenge your explanation if they disagree.
4. Place your cards in order, from smallest on the left to largest on the right. Check that you all agree about the order. Move any cards you need to, until you are all happy with the order.

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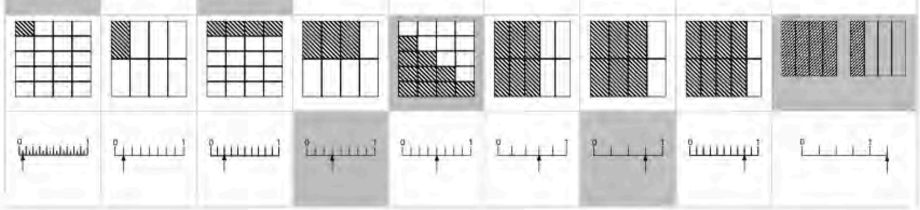
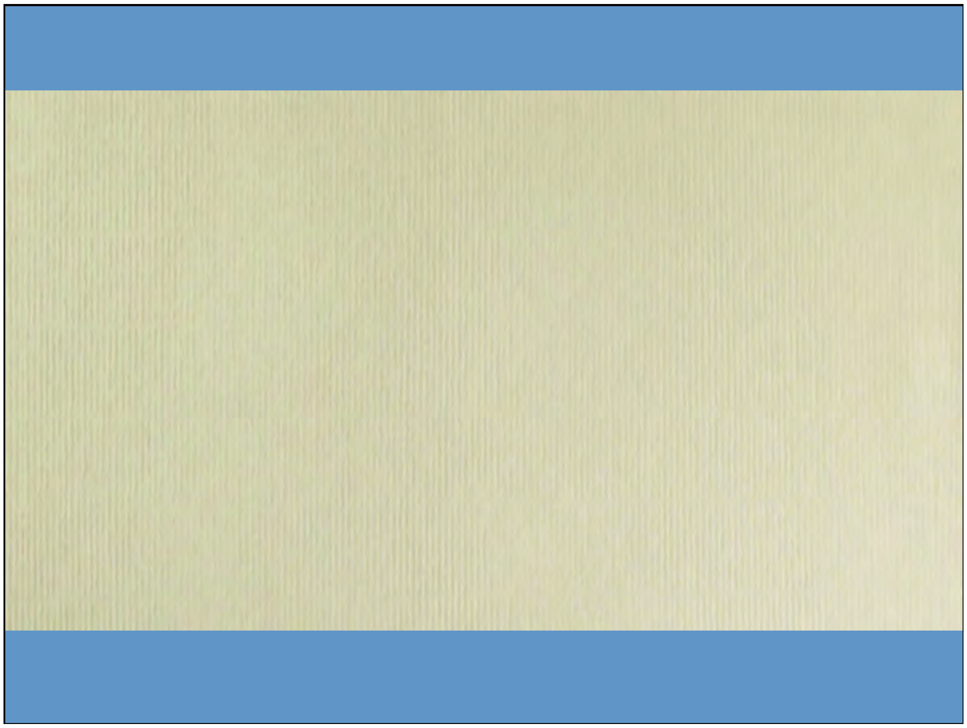
The complete answer set (decimals, %, fractions, area, measure)

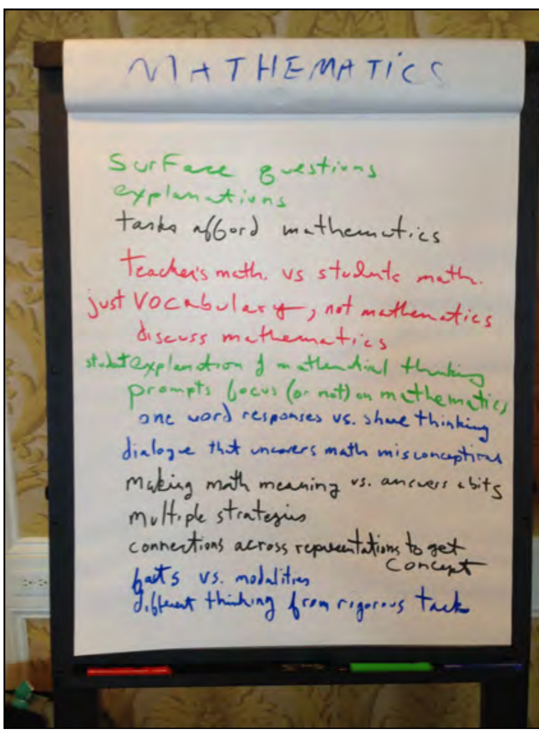
0.05 5%	0.125 12.5%	0.2 20%	0.375 37.5%	0.5 50%	0.6 60%	0.75 75%	0.8 80%	1.25 125%
$\frac{1}{20}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$





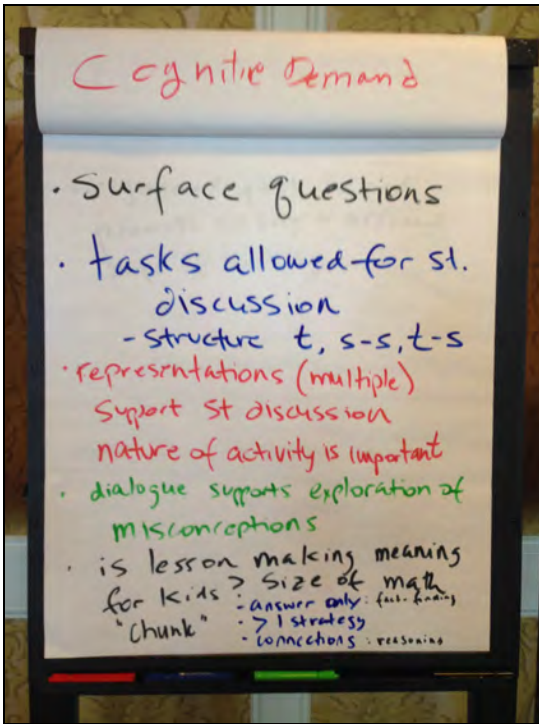
Every time a group looks at videos, there are lots of comments about what the teachers are doing, and what it must feel like to be a student in their classrooms.

And every time, it is easy to organize everything they say into five categories:



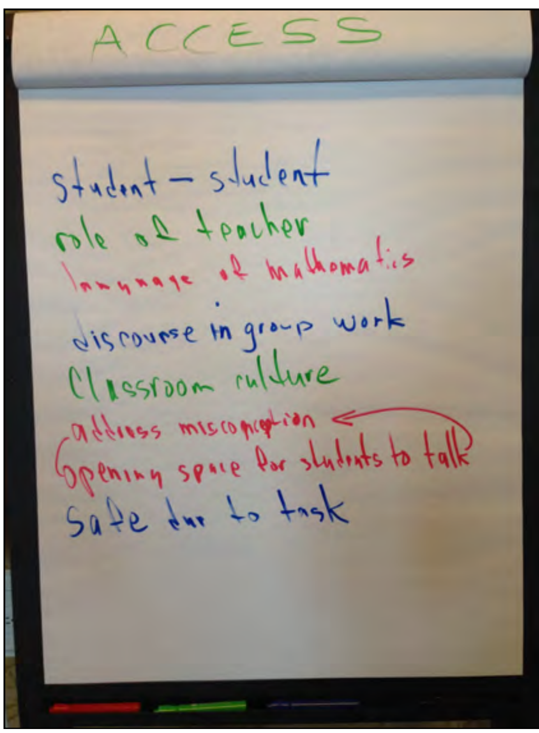
The Mathematics

Is it important, coherent, connected? Where are the big ideas? Are there opportunities for thinking and problem solving?



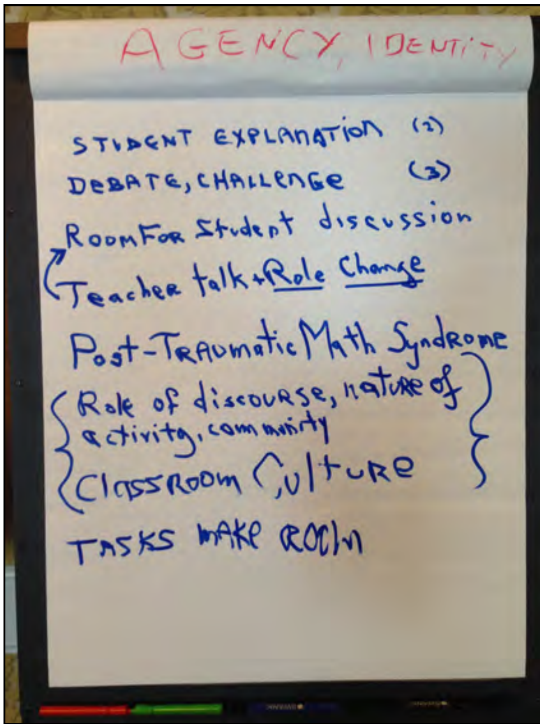
Cognitive Demand

Do the students have opportunities for sense making – for "productive struggle," engaging productively with the mathematics?



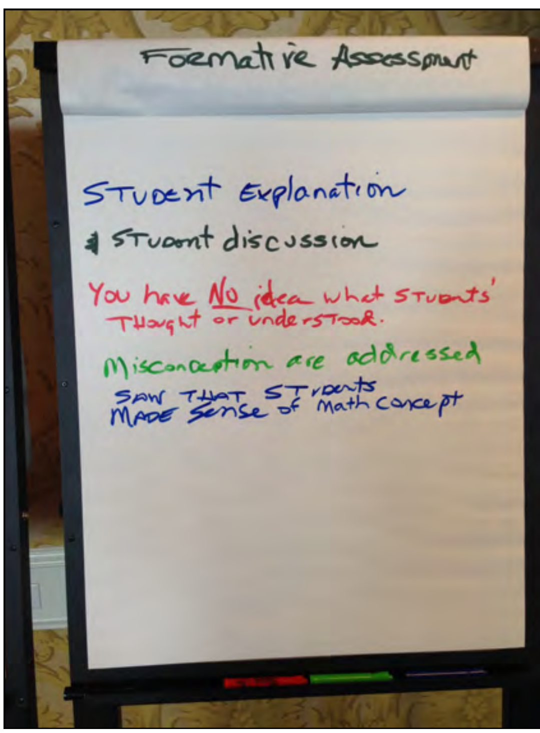
Access and Equity

Who participates, in what ways? Are there opportunities for *every* student to engage in sense making?



Agency and Identity

Do students have the opportunities to do and talk mathematics? Do they come to see themselves as "math people," or people who cannot do mathematics?



Formative Assessment

Does classroom discussion reveal what students understand, so that instruction can be adjusted for purposes of helping students learn?

These are the five dimensions of **T**eaching for **R**obust **U**nderstanding of **M**athematics, or …

– **TRU Math** –

The Five Dimensions of Mathematically Powerful Classrooms

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Formative Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

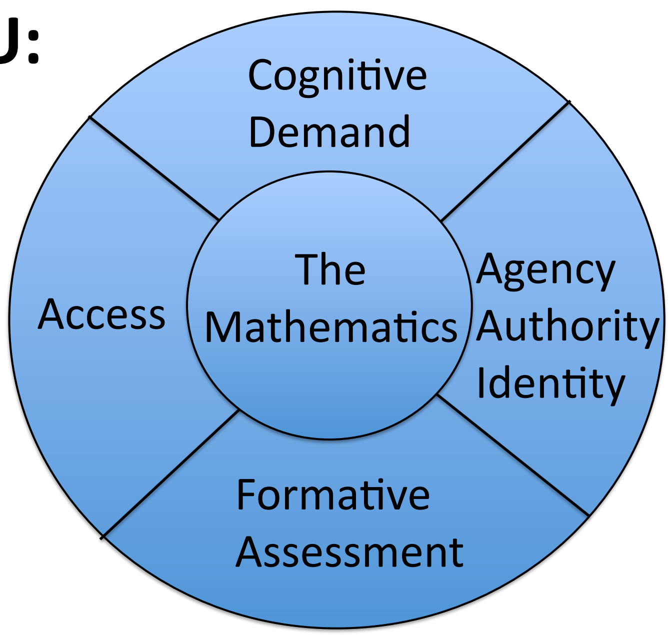
Please note:

Text is linear, while the ideas aren't.

So you might think of it this way,
with the content at the center.

It's all connected.

TRU:



What's New, What's Different?

In a sense, nothing.

That is,

You should recognize and resonate to everything in TRU.

It captures what we know is important. It doesn't offer any "magic bullets" or surprises.

So, What's Different?

TRU is:

- Comprehensive -
- Easy to remember -
- Easy to work on/with -
- It's a natural frame for PD -

Any classroom, from pre-K through graduate school, that does well on these five dimensions, will produce students who are powerful mathematical thinkers.

So much evidence, so little time…

See

http://map.mathshell.org

and

http://ats.berkeley.edu

for evidence, and for the tools I'm about to show you.

Before proceeding, it's ESSENTIAL to understand:

TRU is NOT a tool or set of tools.

TRU is a perspective regarding what counts in instruction, and

TRU provides a language for talking about instruction in powerful ways.

With this understanding, you can make use of any productive tools wisely.

But, we have tools.

(of course.)

TRU contains and aligns with a large set of tools produced by the Mathematics Assessment and the Algebra Teaching Study Projects.

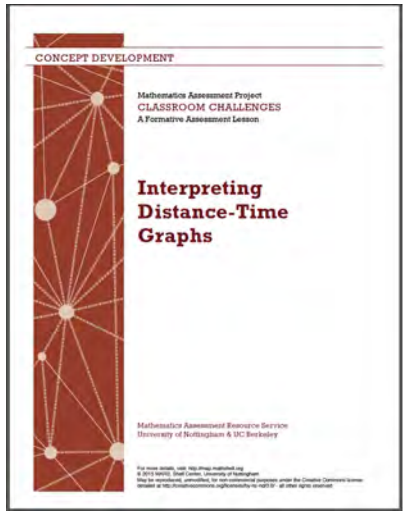
Tools

a. Tools for instruction
b. Tools for planning and reflection
c. Tools for observations

a. Tools for Instruction

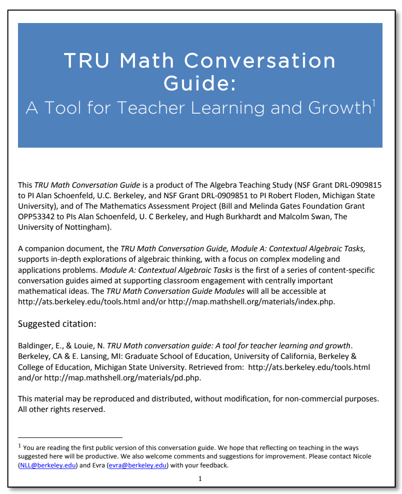
- 100 *Formative Assessment Lessons*
- Support rich student engagement along the TRU dimensions
- More than 5,000,000 downloads
- Strong documented student learning gains
- Download for free at

http://map.mathshell.org/lessons.php



b. Tools for Planning and Reflection

- The TRU Math Conversation Guide.
- The TRU dimensions become arenas for teachers to reflect on their own teaching:
  - in planning,
  - in reflecting on how things have gone
  - in thinking about next steps.



TRU Math Conversation Guide:
A Tool for Teacher Learning and Growth[1]

This *TRU Math Conversation Guide* is a product of The Algebra Teaching Study (NSF Grant DRL-0909815 to PI Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0909851 to PI Robert Floden, Michigan State University), and of The Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP53342 to PIs Alan Schoenfeld, U. C Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham).

A companion document, the *TRU Math Conversation Guide, Module A: Contextual Algebraic Tasks,* supports in-depth explorations of algebraic thinking, with a focus on complex modeling and applications problems. *Module A: Contextual Algebraic Tasks* is the first of a series of content-specific conversation guides aimed at supporting classroom engagement with centrally important mathematical ideas. The *TRU Math Conversation Guide Modules* will all be accessible at http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/index.php.

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[1] You are reading the first public version of this conversation guide. We hope that reflecting on teaching in the ways suggested here will be productive. We also welcome comments and suggestions for improvement. Please contact Nicole (NLL@berkeley.edu) and Evra (evra@berkeley.edu) with your feedback.

1

Start with the core questions:

The Mathematics

How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Cognitive Demand

What opportunities do students have to make their own sense of mathematical ideas?

Access to Mathematical Content

Who does and does not participate in the mathematical work of the class, and how?

Agency, Authority, and Identity

What opportunities do students have to explain their own and respond to each other's mathematical ideas?

Uses of Assessment

What do we know about each student's current mathematical thinking, and how can we build on it?

. . . and expand them.

Before a lesson, you can ask:

- How can I use the five dimensions to enhance my lesson planning?

After a lesson, you can ask:

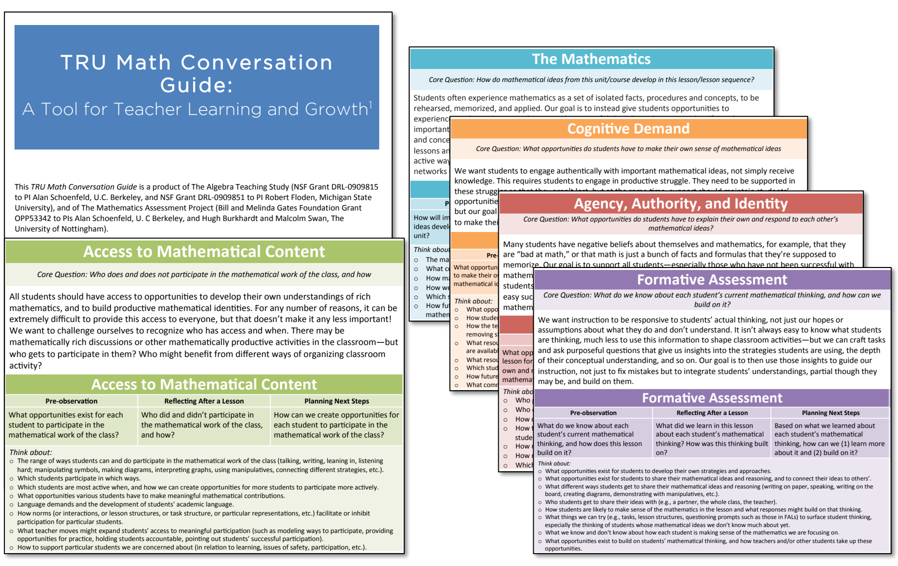
- How well did things go? What can I do better next time?

Planning next Steps, you can ask:

- How can I build on what I've learned?

I'll show you what the conversation guide looks like, and make a quick stop at "access" to illustrate the kind of conversations it's intended to support.

A look at the Conversation Guide



Access to Mathematical Content

Core Question: Who does and does not participate in the mathematical work of the class, and how

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

Access to Mathematical Content

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist for each student to participate in the mathematical work of the class?	Who did and didn't participate in the mathematical work of the class, and how?	How can we create opportunities for each student to participate in the mathematical work of the class?

Think about:

- The range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Language demands and the development of students' academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

Imagine teachers and coaches planning together, watching each other teach, and debriefing using these ideas.

This can be done in lesson study (more below), but, it can also become an ongoing way of thinking about teaching – every day, every class.

c. Ways to Observe Classrooms

Here are two ways of observing:

- Observe as a teacher.
- Observe as a student.

Observe as a teacher

The Mathematics	• Are students learning important mathematics? • Are opportunities made for meaningful connections?
Cognitive Demand	• How long do students spend on each prompt? • Do they engage in productive struggle? • Do teacher questions invite explanations or answers?
Access to Mathematical Content	• Are there multiple ways to get involved productively? • Does the teacher ask a range of students to respond?
Agency, Authority, and Identity	• Who explains most: the teacher or the students? • Do the students give extended explanations?
Formative Assessment	• Does the teacher follow up on student responses? • Does the teacher vary the lesson in the light of student responses?

Observe as if you were a student

The Mathematics	• What's the big mathematical idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Access to Mathematical Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored?
Agency, Authority, and Identity	• Do I get to explain, to present my ideas? Are they built on? • Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	• Do classroom discussions include my thinking? • Does instruction respond to my thinking and help me think more deeply?

Keeping these questions in mind can help you think about your teaching, your observing, or your coaching.

And that's where we cycle back to Lesson Study.

Lesson study is a wonderful process for deep dives into student thinking and learning. Using the TRU Framework names some lesson essentials and provides tools for reflection and refinement.

That's why I said...

Lesson Study and the TRU framework are a marriage made in heaven.

I hope to pursue these issues with you, in partnership, for many years.

Thanks!