Dr. Jorge Prieto Math and Science Academy

Cross-Categorical Diverse Learners, Volume of a Rectangular Prism

Team Members

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Lesson Date:

May 15, 2019

Instructor:

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Grade Level:

5th/6th/7th/8th

1. Title of Lesson

Volume of a Rectangular Prism

2. Research Theme

The long-term goals for our students and how we will get there (theory of action)

Teach scholars to make sense of problems and persevere in solving them by teaching math through problem solving.

Teach scholars to construct viable arguments and critique the reasoning of others through note-taking, board work, and scholar discourse.

3. Background and Research on the Content

In previous years teaching maths to diverse learners in the co-taught and self-contained settings across middle grades it was evident that basic maths facts knowledge limited curriculum opportunities for diverse learners in their current setting (LRE 1), high school (LRE 2), college, and career. In order to adequately address the standards aligned to fifth, sixth, seventh, and eighth grades we began our research deconstructing the Maths Common Core State Standards and progression map for geometry. In September many of the scholars were not able to complete operations involving multi-digit integers, decimal numbers, fractions, or order terms in relation to place value/base ten system which structured the units of study we chose to present to our scholars. Although scholars are in a cross-categorical self-contained setting for their maths instruction, their gaps in maths facts knowledge must be addressed through appropriate grade level standards.

We wanted to present a lesson which maintained rigor, applied grade maths level standards, and provided scholars a real world calculation. Understanding our scholars in this class community are visual and kinesthetic learners we wanted to provide opportunities surrounding composing and decomposing figures to solidify the understanding of properties of shapes. We maximized opportunities to work through problems using models and manipulatives throughout our year together to improve scholar understanding of the properties of shapes, specifically squares and rectangles as they did not possess a solid understanding of this. The Japanese maths Sansu curriculum presented a pace that was appropriate for whole class entry into a concept surrounding geometry. Once scholars developed their knowledge surrounding the properties of shapes scholars moved to the next level of The van Hiele Levels of Geometric Thought to Level 2, Informal Deduction. “Students at Level 2 will be able to follow and appreciate informal deductive argument about shapes and their properties” (Van de Walle, 2010, p. 403).

Scholars were able to learn the maths they may have struggled with in the past, improve their understanding, and practice their strategies on grade level maths through problem solving practice. Scholars built their capacity for discussing their maths calculations through turn and talk opportunities with peers and validated their arguments across all subjects (English Language Arts, Maths, Science, Social Sciences, and Writing) through Claim/Reasoning/Evidence. Our hope is that in future geometry instruction at Prieto and beyond scholars will continue to create claims that serve as proofs to validate whether theorems are true. Van de Walle states, “If you write a theorem on the board and ask students to prove it you have already told them that it is true. If, by contrast, a student makes a statement about a geometric situation that class is exploring, it can be written on the board with a question mark as a conjecture, a question whose truth has not yet been determined.” (Van de Walle, 2010, p. 416). The cross-categorical classroom presents challenges in terms of addressing gaps in knowledge and a strong urgency to provide scholars with remediation surrounding concepts they need to be successful in future maths.

Moving through the levels of geometric thinking was a way for us to remedy scholar capacity in future understanding with the hope that many can attain the general education curriculum in an inclusive classroom in the future or even better yet, the general education classroom without special education support. If we taught the scholars maths focused on memorizing formulas they would not retain the formula nor the understanding to calculate future operations. In a classroom where we feel that we are playing catch up especially for the eighth grade scholars who are operating at a level of geometric thought characteristic of much younger scholars, it was imperative for us to provide them with a solid understanding they could apply to algorithms surrounding parallelograms, and circles in the future. The urgency in our lesson is related to the loss of curriculum opportunities in the self-contained setting which exponentially decreases scholar capacity to calculate grade level maths as they progress through their schooling. Limitations compound making it increasingly difficult for scholars to catch up with same age grade level peers, which can impact their future success in terms of college selection and career.

4. Rationale for the Design of Instruction

- What we learned from studying our own curriculum and other resources
- Why the unit and lesson are designed as they are - for example, why we chose this particular task, representations, contexts, lesson sequence, etc.
- How the unit and lesson design address the research theme

Volume is included in maths Common Core State Standards for fifth-eighth grade scholars, which also afforded an equitable standard based entry point in a lesson that was facilitating learning for several grade levels. In the beginning of the year gaps in scholar knowledge ranged from inability to calculate simple to complex equations surrounding decimals, fractions, order of operations, and calculations that require an algorithm. For scholars who are diverse learners the Sansu Japanese maths curriculum presented a pace that was appropriate for whole class instruction coupled with grade level maths application focused on small group or individual practice problems with guided instruction similar to guided reading. Scholars who attain curriculum through differentiation also need more opportunities to succeed with concepts, Van de Walle agrees and states, “Some students, particularly those with special needs, may need additional structure...In a similar manner, students will find that there is only one solid made of squares - three at each point and six in all - a hexahedron (hex = six), also called a cube” (Van de Walle, 2010, p. 432).

In previous problem solving lessons scholars worked to find the area of rectangles without being introduced to the standard formula. These lessons allowed scholars to begin thinking deeply about area as a concept, and not just as a formula. Through comparisons and discussions of scholar's' ideas, they were able to derive the formula as a class, and use their math to justify the validity of the formula. This experience is the rationale for using a similar teaching through problem solving approach to help scholars learn about volume. “When students develop formulas, they gain conceptual understanding of the ideas and relationships involved and they engage in one of the real processes of doing mathematics”

(Van de Walle, 2010, p.391). Simply teaching formulas does not develop understanding of formulas for scholars, "students can see how all area formulas are related to one idea: length of the base times the height. And students who understand where formulas come from do not see them as mysterious, tend to remember them, and reinforce the idea that mathematics makes sense. Rote use of formulas from a book offers none of these advantages" (Van de Walle, 2010, p. 391).

The properties of cubes and cuboids was necessary for us to present to our scholars because as we facilitated instruction surrounding area through activities such as calculating the area of the classroom it became evident scholars did not have the understanding of what 2d and 3d shapes mean. As we explored three dimensional figures such as cuboids, scholars learned what faces, edges, and vertices meant, prior to that unit of study they were not able to identify faces, edges, or vertices. For our understanding of volume, we also needed to understand these terms. The problem solving skills that scholars will use to make sense of volume is characteristic of the deep understanding and mathematical practices that we hope graduating scholars will be able to apply in high school or beyond, and where remaining scholars will be able to apply to future lessons.

5. Goals of the Unit

The goals of this unit are to develop scholars' conceptual understanding of volume, promote problem solving and equip scholars to articulate their calculations through claim/reasoning/evidence.

Scholars will connect their new learning around volume to their prior knowledge surrounding area, multiplication, and claim/reasoning/evidence as an avenue to present their argument(s). As scholars persevere through constructing nets, filling cuboids with cubes that are 1 - cm^3, and applying the operation of multiplication through the algorithms for area and volume scholars will be able to apply the strategies gained with future challenging maths they encounter.

For Scholars to cement the correlation between the understanding that volume equates to capacity and that the algorithm for volume is secondary.

6. Unit Plan

The lesson sequence of the unit, with the task and learning goal of each lesson. The asterisk (*) shows the research lesson

Lesson	Learning goal(s) and tasks
1	***Lesson Goal:*** How much space do cuboids take up? ***Task*:** Scholars will calculate how much space a large cuboid constructed of tissue boxes takes up. Scholars will be introduced to the term volume.
2	***Lesson Goal:*** What is the volume of a cuboid? ***Task:*** Scholars will solve for volume by filling a cuboid with with cubes that are 1 - cm^3.
3	***RESEARCH LESSON:*** Which has a larger volume a or b? * ***Task:*** Scholars compare the volume of cuboid a.) to cuboid b.) using claim/reasoning/evidence to justify their calculations.
4	***Lesson Goal:*** Develop fluency with the algorithm for volume. ***Task:*** Scholars complete problems surrounding volume algorithm.

7. Relationship of the Unit to the Standards

- How the learning in the unit relates to the grade-level standards.
- How the learning in the unit relates to prior standards and future standards.

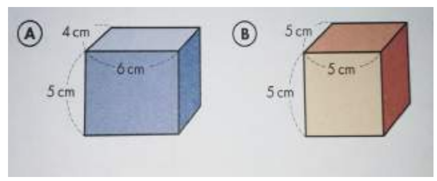
5th/6th/7th/8th Grade Math Standards

Prior learning standards that unit builds on	Learning standards for this unit	Later standards for which this unit is a foundation
4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. CCSS.MATH.CONTENT.5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	CCSS.MATH.CONTENT.5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. CCSS.MATH.CONTENT.7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	CCSS.MATH.CONTENT.6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. CCSS.MATH.CONTENT.8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. CCSS.MATH.CONTENT.HSG.GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

8. Goals of the Research Lesson

Scholars will be able to compare the size of two cuboids, articulate a claim identifying which cuboid is larger, provide reasoning for their claim, and justify their claim through evidence using the algorithm for Volume. Scholars will understand that length, width, and height are used to calculate volume using the operation of multiplication.

9. Research Lesson Plan

Learning task and activities, anticipated student responses, key questions or comparisons that will build insights	Teacher Support	Assessment (Points to Notice)
Introduction We have constructed cuboids out of paper and figured out how many cubes that are 1 - cm³ we can fill a cuboid with earlier this week. Now let's see how we can use volume to compare the size of two cuboids. **< Getting Started >** Which of these two cuboids has a larger volume? Scholars think about ways to find the volume of the cuboids without using cubes that are 1 - cm³. Scholars are asked to share their prediction with peers during Turn and Talk #1.		What reasons are scholars attributing to their choice of cuboid A or cuboid B? Are calculations being discussed?
Posing the Task How do mathematicians use volume to compare the size of two cuboids? ¿Cómo usan los matemáticos el volumen para comparar el tamaño de dos cuboides? ***<Problem>*** Yesterday we used cubes that are 1 cm³ to fill a cuboid to find the amount of space cubes and cuboids take up. Which of these two cubes is larger? Use the volume of the cuboids to justify your claim.	Teacher will pose Guiding Question/Pregunta Orientadora verbally and visually. Scholars have cubes that are 1 - cm³ at their tables.	Scholars will write these questions. Scholars will write the problem and begin to assess the way they are going to solve this problem.

Anticipated Student Responses S1: Student use models of the two cuboids to compare. Student fills each model with cubes that are 1 cm^3 and count the number of cubes required to fill each to determine volume. Student finds that cuboid A has a volume of 120 cm and cuboid B has a volume of 125 cm S2: Student uses models of the two cuboids, find the number of unit cubes it would take to fill the bottom layer, and then use repeated addition to count how many cubes would fill the whole cuboid. A: 24+24+24+24+24=120 B: 25+25+25+25+25=125 S3: Student finds the number of unit cubes it takes to fill the bottom layer and instead of using repeated addition, multiplies the number of unit cubes in the bottom layer times the number of layers. A:24 x 5 = 120 B:25 x 5 = 125 S4: Student uses multiplication to find the number of cubes that it would take to fill the bottom layer, and then multiplies that product by the number of layers. A: 4 x 6 x 5 = 120 B: 5 x 5 x 5 = 125	Teacher will support student problem solving and will move across the classroom observing scholar responses.	Are scholars creating models in their journals? Are scholars using 1 cm^3 cubes to fill the cuboids? Do claims/ reasoning identify 1 cm^3 cubes as a rationale? Was there discussion surrounding why scholars multiplied the bottom layer? Does the scholar's reasoning offer justification aside from the three integers being multiplied?
Comparing and Discussing, including Teacher Key Questions Students compare ideas in groups, then as a class. Teacher facilitates comparison and discussion	Teacher will facilitate peer discussion and peers will share their claim/reasoning/evidence to class at large.	

of student ideas to help them progress from direct modeling by placing the cubes in the cuboids to using mathematics to determine volume. Suggested order for comparing student ideas: S1: Have student that used direct modeling approach for determining volume of each cuboid explain their idea and share their results. The results of this model can be used to verify the validity of student that used mathematical models to find the volume. S2: and S3: Next, scholars should discuss the connection between S2 and S3's idea. How both scholars find the number of cubes in just one layer, and then used either addition or multiplication to find the volume of each cuboid to justify their claim of cuboid A being larger than cuboid B. Finally, have S4 explain their idea. Have scholars discuss the similarity between S3 and S4, specifically how both of them multiplied the volume of one layer by the number of layers, and how S3 made a model to find the volume of the one layer, while S4 used multiplication to find the volume of one layer.		
Summing Up **<Summary>** To find the volume of a cuboid or rectangular prism we can find the volume of the bottom layer, and then multiply it by the number of layers to find the total volume of a cuboid. **<Reflection>** Today I learned from _______ that _______	Teacher will solidify learning and ask scholars to reflect on what they learned in today's lesson.	

10. Points of Evaluation (Assessment)

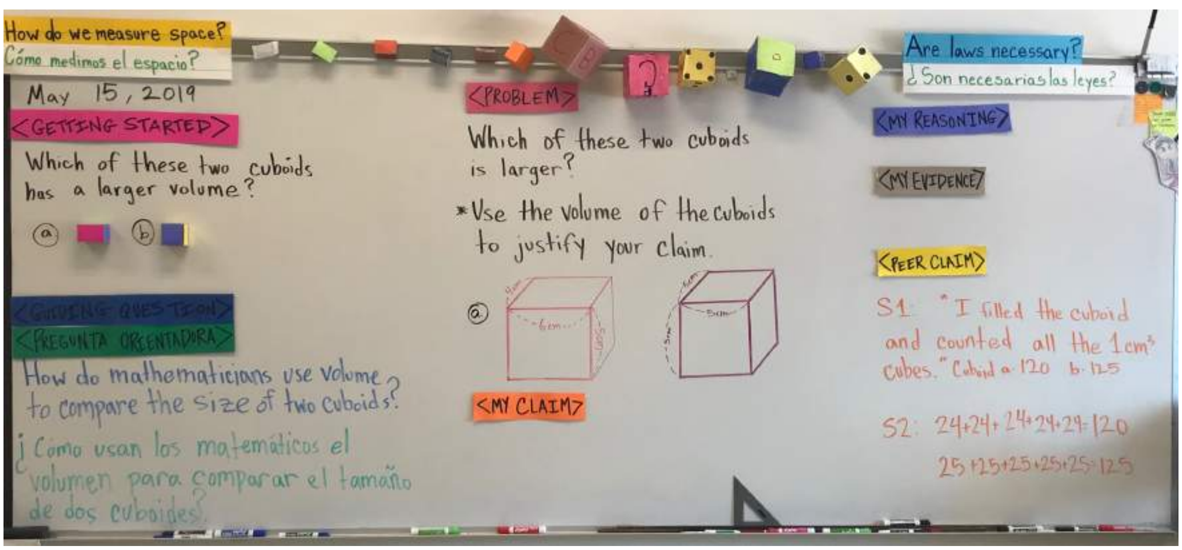
Prompts to focus observation and data collection.

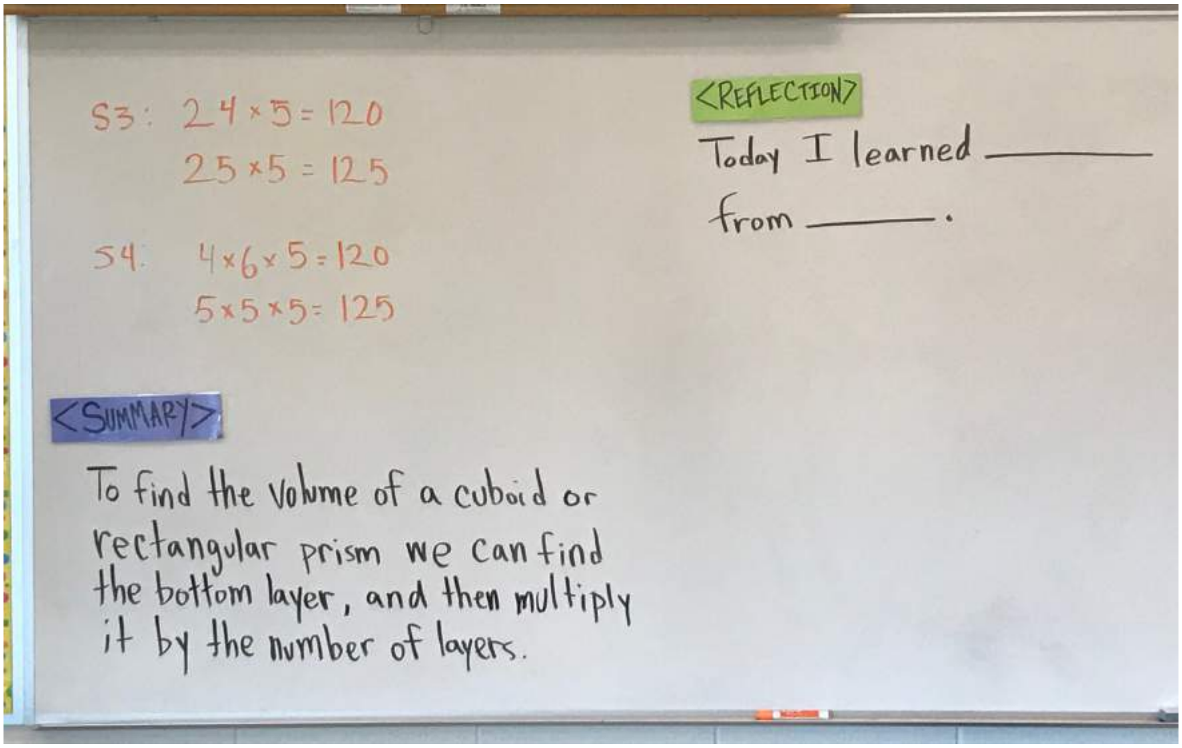
Are scholars able to justify their claim with evidence?

Does the comparison and discussion of student ideas move scholars understanding towards the goal of the lesson as articulated in the guiding question?

How do scholars make their understanding known using models, notebooks, and verbal responses?

11. Board Plan





12. End of Cycle Reflection

What Did We Learn? (to be filled out after the post-lesson discussion)

13. Reference

De Walle, V. (2010) *Elementary and Middle School Mathematics: Teaching Developmentally 7th Edition***.** Boston, MA: Ally and Bacon.