

Grade 6 Mathematics Lesson Plan

Date: June 16, 2017, 5th Period

Location: Grade 6 classroom

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1 Name of the Unit: What calculation is it?

2 Goals of the Unit

- (1) Students will understand the meaning and methods of calculation when the multiplier is a fraction. Students also understand that the properties of multiplication still hold even when the multiplier is a fraction and further develop their ability to use them. (For “Multiplication of Fractions” section.)
- (2) Students will understand the meaning and methods of calculation when the divisor is a fraction and extend their ability to use it. (For “Division of Fractions” section.)
- (3) Students will further develop their ability to select the appropriate operation to solve problems by solving application problems involving multiplication and division of fractions.**

3 About the Unit

(1) About Mathematics

The research lesson is positioned after students learned multiplication and division of fractions. Since students learned multiplication and division of decimal numbers, they are a little confused about the size relationships of numbers. This is due to the fact that they have to modify their beliefs “multiplication makes a number bigger” and “division makes a number smaller.” As students study multiplication and division of fractions in this unit, students struggle to make sense of what it means to multiply or divide by fractions.

Double number line diagrams allow students to visually grasp the relationships among numbers involved in multiplication and division of fractions. By mastering the ability to represent problem situations in double number line diagrams, we believe students can grasp the size relationships among numbers in multiplication/division situations. Moreover, by grasping the relationships among the numbers in problem situations, students can further develop their ability to select appropriate operations to solve problems.

(2) About students

In general, the students in the class tend to be not fond of mathematics or lack confidence in their ability in mathematics. During the independent problem solving time, they try their best to solve the assigned problem. However, only a few students regularly raise their hands to volunteer their ideas, and even they have difficulty explaining their ideas in a way that may be easy for their classmates to understand. On the other hand, if they do not understand a mathematical procedure or how to solve a problem, they ask either their classmates or the classroom teacher. Thus, they seem to have sincere desire to understand.

In the study of multiplication and division, many students already knew the procedures for calculation, but they did not know why the procedures work. Therefore, we spent time to help them understand the meaning of multiplying or dividing by a fraction. While studying multiplication and division of fractions, each step of calculation process was connected to equations with words and double number line diagrams. In particular, double number line diagrams are extensively used as students worked with word problems, and many students are now able to represent problem situations using double number line diagrams.

(3) About Instruction

In this lesson, we want students to understand the usefulness of double number line diagrams for grasping the size relationships of numbers in the problem situations, estimating the answer and deciding on the appropriate operation. We also want students to experience the usefulness of double number lines as a communication tool. Moreover, by actually using double number line diagrams, we want students to enhance their problem solving ability.

The following hypotheses have been considered for this lesson.

- 1) If students can represent problem situations using double number line diagrams, they can grasp the relationships of the numbers and quantities in the situations and estimate the answers.
- 2) If students are given opportunities to communicate each other's ideas in small groups, they can incorporate their friends' ideas into their own as well as feel more confident about their ideas.
- 3) After learning about how to represent problem situations using double number lines, if they can solve different problems by making use of diagrams, they can experience the sense of achievement and satisfaction.

4 Vision of ideal students and teaching strategies to approach the vision

(1) To help students understand the mathematical purpose of a task and approach mathematics learning purposefully

i. Construction of the unit plan

When multiplication and division by fractions are introduced, we made intentional references to multiplication and division by decimal numbers so that students might be able to make use of their prior knowledge. Moreover, by converting decimal numbers to fractions, we tried to make it easier for students to hypothesize that the properties of operations would hold even with fractions. Furthermore, in order to help students develop more concrete image of division by fractions, we added a period in the unit so that we can pull out of division by fraction from the study of Fraction \div Whole Number.

ii. Strategy to present the learning task (in this lesson)

Even though students will be choosing a problem to solve, by showing 3 word problems together, students can attend to the differences in the way the problems are represented in double number line diagrams. Moreover, students can see that double number line

diagrams allow them to observe the relationships among numbers by simply changing the location of the unknown (\square).

(2) To help students become able to add/correct/improve their ideas based on collaboration with other students

i. Communication among students (in this lesson)

Within the problem-solving based lesson, we will set up time for students to share their ideas in small groups. In this way, students can gain more confidence in their own ideas or modify their ideas and representations based on their friends' ideas. Moreover, students will have more opportunities to explain their ideas verbally.

(3) To help students who can represent and summarize their ideas

i. Set up time for students to draw double number line diagrams or other representations

Throughout this unit, as we investigated word problems and calculation processes, we constantly included double number line diagrams. In addition, by constructing the initial part of double number line diagrams with the teacher, students gradually developed the ability to draw diagrams on their own.

ii. Set up times for independent problem solving and small group discussion

Throughout the unit, we tried to have about 7 – 10 minutes each for independent problem solving and for small group discussion. In addition, in some lessons, students used small white board to present their ideas to the class so that whole class can discuss their ideas.

iii. Set up time for applying what students have learned (in this lesson)

We want to make sure that students have the opportunity to see if they can use double number line diagrams to select an appropriate operation on their own. Moreover, by assessing each student in this activity, teachers can provide further support students need based on their achievement level.

5 Unit Plan (total of 21 lessons)

Multiplication of Fractions	L1 Setting up Fraction \times Fraction	<ul style="list-style-type: none"> Think about the problem situations using their prior learning of multiplication of decimal numbers and proportional relationships.
	L2 Ways to calculate $\frac{4}{5} \times \frac{2}{3}$ Part 1	<ul style="list-style-type: none"> Making connections to diagrams, help students understand the meaning of multiplying by $\frac{2}{3}$ as dividing $\frac{4}{5}$ into 3 equal parts then taking 2 of the parts.
	L3 Ways to calculate $\frac{4}{5} \times \frac{2}{3}$ Part 2	<ul style="list-style-type: none"> Calculate using the property of multiplication by changing the multiplier into a whole number then adjust the product as we did with multiplication of decimal numbers.
	L4 \times (Proper Fraction), \times (Mixed Number) and calculation of 3 numbers	<ul style="list-style-type: none"> Calculate by changing mixed numbers into improper fractions. Simplify before calculating the final product.

	L5 Area of rectangles with fractional dimensions	<ul style="list-style-type: none"> Help students understand the meaning of area with fractional dimensions. Identify unit fractions.
	L6 Volume of boxes with fractional dimensions	<ul style="list-style-type: none"> Apply the idea of unit fractions in volume of boxes. Apply also to other 3-D shapes.
	L7 Applications of commutative, associative, and distributive properties	<ul style="list-style-type: none"> Verify that these properties still hold even when fractions are involved by replacing whole numbers by fractions.
	L8 Meaning of reciprocals and ways to find them	<ul style="list-style-type: none"> Find pairs of numbers whose product is 1.
	L9 Applications	<ul style="list-style-type: none"> Identify mistakes in the given solutions. Explain the relationship between the size of the multiplier and the product. Explain the steps of calculation.
Multiplication of Fractions	L1 Fraction \div Whole Number ($\div 2, \div 3$)	<ul style="list-style-type: none"> Students understand the meaning of division by fractions using double number line diagrams and area diagrams.
	L2 Fraction \div Whole Number ($\frac{3}{4} \div 5$)	<ul style="list-style-type: none"> Figure out quotients by transforming expressions.
	L3 Simple cases of \div Fractions – meaning of division is clear	<ul style="list-style-type: none"> Select numbers for \square in $\square \div \frac{1}{4}$ that can be figured out and think about ways to determine the quotients.
	L4 Meaning of $\frac{2}{5} \div \frac{1}{4} = \frac{2}{5} \times 4$	<ul style="list-style-type: none"> Think about the meaning of $\frac{2}{5} \div \frac{1}{4}$ using double number line diagrams and transformations of expressions.
	L5 Meaning of $\frac{5}{6} \div \frac{2}{3}$ and its calculation	<ul style="list-style-type: none"> From word problems, set up the correct calculation and find the answers using figures and double number line diagrams.
	L6 Simplifying calculations, Whole Number \div Fractions, calculations with mixed numbers	<ul style="list-style-type: none"> Developing fluency of calculation and integrating division.
	L7 Mixed numbers and decimal numbers \div Fractions	<ul style="list-style-type: none"> Developing fluency of calculation and integrating division.
	L8 Size relationship between the dividend and the quotient	<ul style="list-style-type: none"> Understand the size relationship between the dividend and the quotient based on double number line diagrams and contexts of the problems.
	L9 Calculations with 3 numbers	<ul style="list-style-type: none"> Calculate expressions that involve both multiplication and division operations.
	L10 Times as much and division	<ul style="list-style-type: none"> From word problems, determine how many times as much or the base quantities.
	L11 Selecting appropriate division calculation	<ul style="list-style-type: none"> Write 2 distinct problems from the same situation and set up the correct expressions for them.
Application of \times and \div	L1 [Research Lesson] Selecting appropriate operation	<ul style="list-style-type: none"> Match the problem situations with the given expressions and explain their reasoning using double number line diagrams and/or equations with words.

6 Research Lesson

- (1) Goal: Students can find the appropriate calculation from double number line diagrams and also generate appropriate problem situations that match the given double number line diagrams.

(2) Flow of the Lesson

See attached