Teaching Math
through Problem-Solving (TMtPS)

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Common Core Standards for Mathematical Practice

- 1 Understand problems and persevere in solving them

"Problem solving must be the focus of school mathematics…"

NCTM *Agenda for Action*, 1980

What does "problem" mean?

"Problem solving means engaging in a task for which the solution is not known in advance."
NCTM, *Principles and Standards for School Mathematics*
(Washington, DC: NCTM, 2000)

"I do, we do, you do" is antithetical to developing perseverance and problem solving.

A Japanese perspective: Three levels of teaching

- Level 1: *Explanation* The teacher can tell students the important ideas of mathematics such as facts, concepts, and procedures.

- Level 2: *Explanation and rationalization* The teacher can explain the meanings and reasons of the important ideas of mathematics so that students can understand them.

- Level 3: *Student-centered exploration* The teacher can provide students opportunities to discover and understand the important ideas, and support their learning so that the students become independent learners.
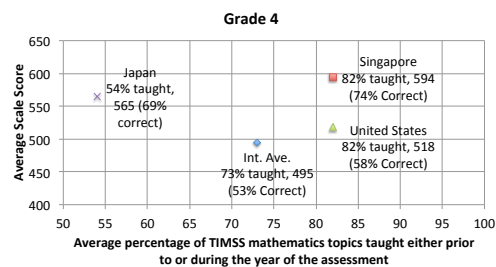
Sugiyama 2008

Level 3 teaching: Teaching mathematics through problem solving
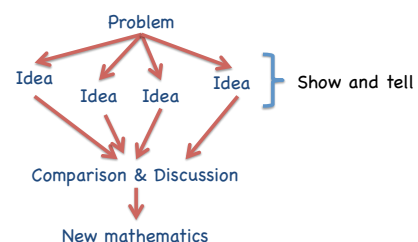
But how will they do on the test?

Average percentage of TIMSS math topics taught in school vs. math achievement (average scale score) on the TIMSS 2003

Source TIMSS 2003 International Mathematics Report
Grade 8: Exhibit 5.7 (p.192), Exhibit C. 1 (p.400)
Grade 4: Exhibit 5.7 (p.193), Exhibit C. 1 (p.402)

Let's look at "teaching through problem solving" through some specific content
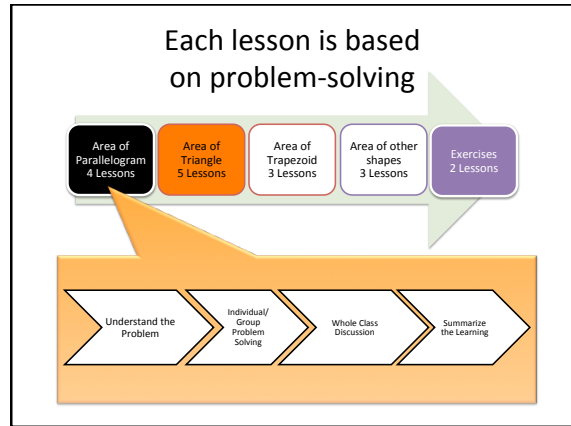

Common Core, grade 6

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms.
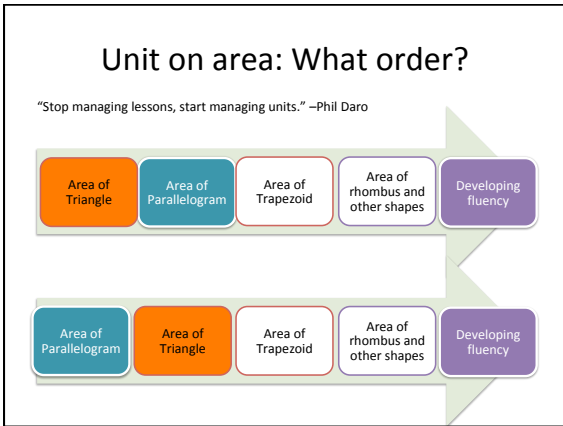

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Teaching guide for the Japanese Course of Study, grade 5

The main objectives are to determine how to find the areas of triangles, parallelograms, rhombuses, and trapezoids based on the methods already learned for determining the areas of geometrical figures, to explain the methods, to create formulas, and to cultivate the ability to think logically in the process. Students are expected to explain how to determine area based on the ideas such as the following:

1) Move part of the geometrical figure and transform it to an already-learned shape of equivalent area
2) Think about the given shape as a half of the area of an already-learned geometrical figure
3) Decompose a shape into several already-learned geometrical figures

Unit on area: What order?

"Stop managing lessons, start managing units." –Phil Daro

Each lesson is based on problem-solving

Lesson 1

"Draw and compare the areas of many different figures with perimeter 24 cm."

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1 Think about how to find the area of parallelogram ABCD below.

Explain the two students' ideas below.

Kaori moved triangle ECD to change parallelogram ABCD into rectangle FBCE to find the area.

Why did Kaori change the shape into a rectangle?

Hiroki moved quadrilateral GKCD to change parallelogram ABCD into ...

The area of a parallelogram can be found if by changing it into a rectangle.

Even though the shapes are changed, the area stays the same, doesn't it?
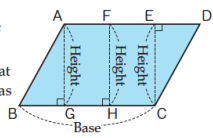
Lesson 2

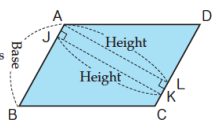2 Based on Kaori's idea on the previous page, calculate the area of parallelogram ABCD on the right.

Let's come up with a formula to find the area of a parallelogram.

The area of the parallelogram above is equal to the area of a rectangle with what dimensions?
Which parts of the parallelogram are equal to the length and the width of this rectangle?

3

Slide 1

In the parallelogram on the right, if we make side BC as the **base**, the length of the segments that are perpendicular to the base such as segment EC is called the **height** of the parallelogram.

If we consider side AB as the base, the height will be the length of the segments such as those shown on the right.

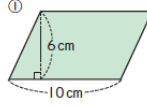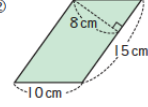⭐2 Calculate the area of the parallelogram above by considering side BC as the base.

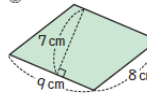The area of a parallelogram can be calculated using the following formula. **Summary**

Area of parallelogram = Base × Height

Slide 2
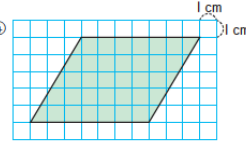
Lesson 2, cont. (practice)

▶2 Calculate the area of the parallelograms below.

① 6 cm — 10 cm —
② 8 cm, 15 cm — 10 cm —
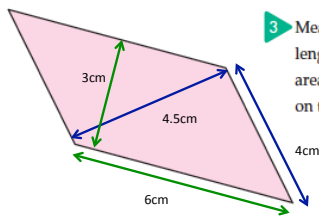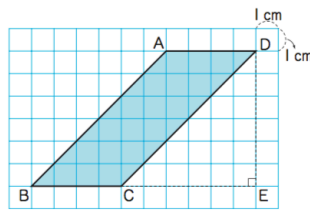③ 7 cm, 9 cm, 8 cm
④ 1 cm, 1 cm

Slide 3

Lesson 2, cont. (practice)

▶3 Measure the necessary lengths and calculate the area of the parallelogram on the left.
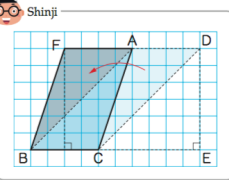
3cm, 4.5cm, 4cm, 6cm

Slide 4

Lesson 3

3 Think about how to find the area of parallelogram ABCD on the right when side BC is considered as the base.

The height is … Kaori

1 cm, 1 cm

Slide 5

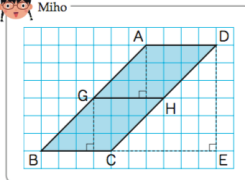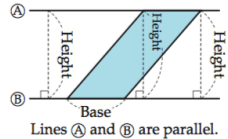⭐1 Explain the two students' ideas using the diagrams.

Shinji — Miho

⭐2 Based on the two students' ideas, calculate the area of parallelogram ABCD.

Slide 6

As shown on the right, sometimes the height of a parallelogram can be found outside of the figure.

Lines Ⓐ and Ⓑ are parallel.

⭐3 If BC is the base, what is the height in cm?

⭐4 Calculate the area to see if the formula on page 31 still applies. Use BC as the base and the appropriate height.
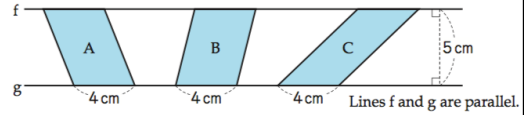
Lesson 3, cont. (practice)

4 ▶ Calculate the area of the parallelograms below.

① 5.5 cm 4 cm

② 6 cm 8 cm 3 cm

③ 4 m 2 m 5 m

④ 1 cm
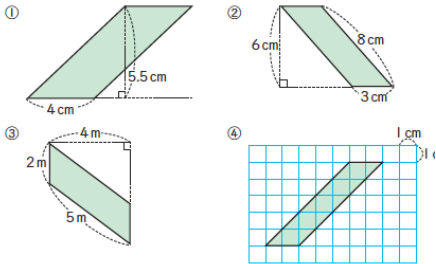
Lesson 4

4 Find the area of parallelograms A, B, and C below.

f

A B C 5 cm

g 4 cm 4 cm 4 cm Lines f and g are parallel.

⭐ Explain why the area of parallelograms A, B, and C are equal.

Summary

Even if parallelograms are shaped differently, if the lengths of their bases and heights are equal, their areas will also be equal.

Lesson 4, cont. (practice)

5 ▶ What is the area of parallelogram E?

The area of parallelogram D is 4cm², and lines k and l are parallel.

k

D (4 cm²) E

l 2 cm 1 cm