# RESEARCH LESSON PROPOSAL FOR 2ND GRADE



For the lesson at the Chicago Lesson Study Conference on May 15-16, 2019

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**Title of the Lesson: Recognizing Equal Group Situations**

**1. Brief description of the lesson:**

Students will be shown a picture of an amusement park. Students will be asked what they notice about the amusement park. Through discussion, students will describe the number of children and the number of groups on each ride. Then, students will be asked what all of the rides have in common. They will notice that there are the same number of children in each group (equal groups). Then students will be asked which ride is different. Students will discover that there is not the same number of children in each group on the ferris ride (unequal groups). At the end of the lesson, students will discover that equal groups are special because there is the same number in each group.

**Unit goals**

- Students will deepen their understanding that when objects are arranged in equal-sized groups, the situation can be represented using a multiplication math sentence. Students will recognize the benefit of multiplication and use multiplication when they need to find the total number of objects.
    - Students reason about and express ways to construct multiplication tables (facts) based on ideas such as repeated addition and the relationship between multipliers and products.
    - Students will understand that the first number in a multiplication math sentence signifies the group size, or multiplicand, and the second number in the math sentence indicates the number of groups, or multiplier.
    - Using pictures, diagrams, words, and math sentences, student represent problem situations that can be solved using multiplication.
    - Students will understand that multiplication may be used to represent comparison situations limited to length measurements (N times as long).
    - Students will understand that when a number can be reached by counting by 2s or decompose into two equal numbers, it is called an even number, and those numbers that are not even numbers are called odd numbers.
    - Students construct and recite fluently the multiplication tables for 5, 2, 3, and 4.
    - Students will understand problem situations that involve multiplication and demonstrate understanding of the multiplication tables and the meaning of multiplication.
    - Students understand the properties of multiplication, such as how products increase by the multiplicand when the multiplier increases by one, and the commutative property of multiplication.

## 2. Research Theme

***Construct viable arguments and critique the reasoning of others.***

Students to be able to use mathematical language and models to describe and justify their thinking, as well as respond to, add on to, and critique the strategies of their peers.

## 3. Lesson goals

- Students will recognize when objects are organized in equal and unequal groups.
- Students will explain what makes equal groups different from unequal groups.
- Students will explain the usefulness of equal group situations to count more easily.

## 4. Relationship of the Unit to the Standards:

| Domain | Code | Standard for Mathematical Content |
|---|---|---|
| Operations and Algebraic Thinking | 2.OA.3 | Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. |
| | 2.OA.4 | Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. |
| | 3.OA.1 | Interpret products of whole numbers, e.g., interpret 5x7 as the total number of objects in 5 groups or 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5x7.* |
| Geometry | 2.G.2 | Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. |

In this unit, students will be introduced to multiplication for the first time. In kindergarten and first grade, students have had foundational experience with the ideas of equal groups. For example, kindergartners learned how to skip-count by 2s, 5s and 10s (*CCSS.MATH.CONTENT.K.CC.A.1*). In first grade, students had a lot of opportunity to skip-count within 120 (*CCSS.MATH.CONTENT.1.NBT.A.1*).

Although the Common Core State Standards does not directly discuss multiplication in grade 2, this unit does address multiple second grade standards. For example, this unit directly addresses standard *CCSS.MATH.CONTENT.2.OA.C.3: Determine whether a group of objects (up*

*to 20) has an odd or even number of members e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addend.* Instead of writing, say, 6+6=12 to show a number is even, students can express that by writing 2x6. This will be the 5th lesson in the unit.

This unit also relates to standard *CCSS.MATH.CONTENT.2.OA.C.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.* Students begin the unit by recognizing objects arranged in equal groups, then representing equal-group situations with multiplication sentences, and arranging counters to match a multiplication sentence. In lessons 4 and 5, students see counters and objects arranged in rectangular arrays and write addition and multiplication sentences to express the situations.

Finally, this unit relates to the third grade standard *CCSS.MATH.CONTENT.3.OA.A.1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each*. For example, students will describe a context in which a total number of objects can be expressed as 5 × 7. It is important to note, however, that in this unit we interpret 5x7 as 5 being the number of objects in each group, and 7 being the number of groups.

## 5. Background and Rationale

**<u>Rationale for choosing our research theme:</u>**

**Research Theme: *Construct viable arguments and critique the reasoning of others.***

*Students to be able to use mathematical language to describe and justify their thinking, as well as respond to, add on to, and critique the strategies of their peers.*

The research theme for the unit and lesson comes from Common Core State Standard for Mathematical Practice 3: construct viable arguments and critique the reasoning of others. Our school-wide research theme was chosen because, as a school, we want to focus on students developing the skills to defend their math conclusions, discuss their math with peers, and question or critique their peers' math or even their own math.

This is an excerpt from our research theme rationale from previous years:

> *"With our transition to learning through problem-solving and the Sansu math curriculum, we saw that there was a great need for students to improve their ability to explain their reasoning using mathematical language or models. Through math journals and problem-solving lessons, we expect that students will improve in providing justification for their own thinking, and be able to question, critique, and respond to their peers' math. We wanted to further expand and develop our ability to facilitate*

*mathematical discussion among our students. This will ultimately achieve our goals of helping students verbalize their ideas, develop mathematical vocabulary, and foster a trusting learning community."*

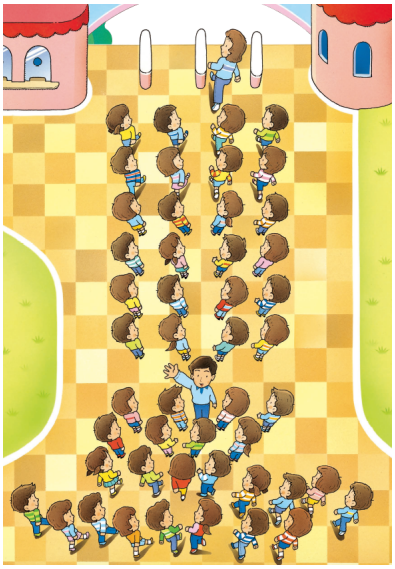
Chavez has now been using the Sansu/Japan Math curriculum in primary grades for four years. We have noticed that our students have exceeded our expectations for having mathematical conversations, however, there are aspects we'd like to improve. Most of our higher-achieving students have eagerly grasped onto math discussions, but we want to ensure that each child has an entry point into the conversation. Given that the majority of our students are ELLs, we are always trying to find new ways to introduce vocabulary and encourage a higher level of discussion. We've noticed that greater alignment and adherence to the Sansu/Japan Math curriculum and teacher commitment to teaching with problem solving allow our students to have a deeper understanding around math.

**Rationale for teaching our unit topic:**

In previous years, we have used the Sansu curriculum to teach the beginning part of the multiplication unit. We have never gone beyond teaching the concept of multiplication. For example, our past students have understood that equal groups make objects easier to count, repeated addition is used to solve a multiplication math sentence, and *number in each group x number of groups = total number*. Although using multiplication and division is not a standard until 3rd grade, the curriculum has always moved into multiplication fact fluency. We have always run out of time in the year to teach those units. In addition, the 2nd grade standard states that students will, "work with equal groups of objects to gain foundations for multiplication."

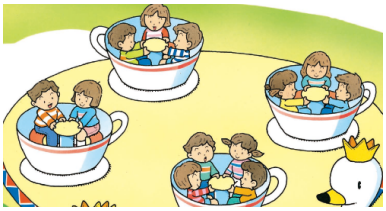
One reason we have chosen to teach this unit topic is to explore the differences between between the Japan Math curriculum and the Sansu curriculum. Another reason is because 2nd grade students are eager to learn multiplication. They are always asking, "When will we do times?" Many of them have been introduced to multiplication in the computer program STMath. Now that they are fluently adding and subtracting, we think they will be ready and eager to see the usefulness of using multiplication to represent equal group situations.

1) Japan Math eliminates the "line" problem that we found especially helpful for students to differentiate between equal and unequal groups. We have decided to include this problem in our unit.

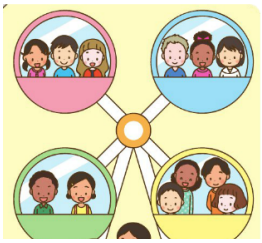


*Example of line problem*

2) The "amusement park" problem has changed. Sansu has the children in the teacups in unequal groups, whereas Japan Math has the number of children in the ferris wheel in unequal groups. We think the children in the ferris wheel are easier to see. We are interested to see if it's easier for students to notice how the groups on the ferris wheel differ from the other groups.



*Sansu*



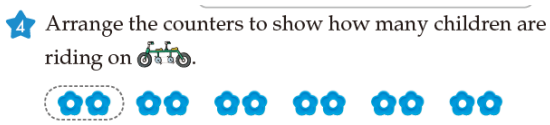
*Japan Math*

The amusement park page is also just one page, with students to write their ideas on the adjacent page. We like how it gives space for students to express independently how many children are on each ride.


*Sansu*


*Japan Math*

3) The Sansu curriculum emphasizes the use of counters for students to represent the children riding on bicycles.




*Sansu*

Meanwhile, the Japan Math curriculum does not mention students using counters to represent the number of children in the first lesson. JM first has students show multiplication situations using counters in Lesson 3.

4) Japan Math has added Lesson 5: "Understand even and odd numbers".
We are eager to see how those differences impact our students' understanding of multiplication.

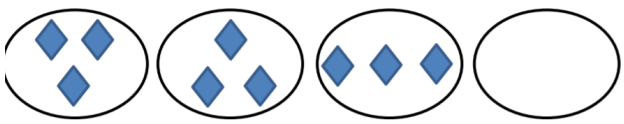
## 6. Research

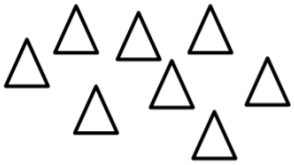
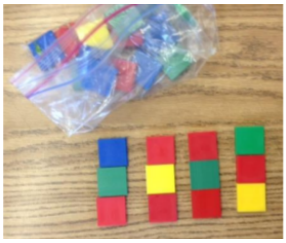
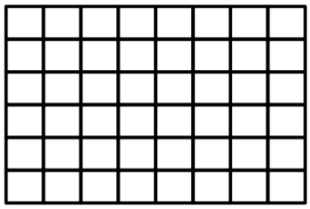
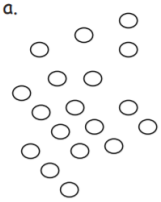
We researched the development of multiplication across primary grades and learned that most curricula begin teaching the concept by introducing equal groups. After students understand that equal groups must have the same number, they progress to learning repeated addition and eventually arrays. The multiplication symbol (x) and the idea of a new operation (multiplication) are not typically introduced until third grade.

We investigated the EngageNY math curriculum, which is developed and maintained by the New York State Education Department (NYSED. We found that students are first exposed to the concept of multiplication in the middle of grade 2 in the following progression:



Students' understanding of multiplication is scaffolded through the support of pictures, sentence stems, and manipulatives. Below are examples of student work from each topic:

| | |
|---|---|
| **Topic A: Equal Groups** | Draw 1 more equal group.  |

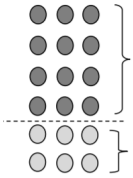
<table>
<tr><td></td><td>  <br>_____ + _____ + _____ + _____ = _____<br><br>4 groups of 2 = _____</td></tr>
<tr><td>Topic B: Arrays and Equal Groups</td><td>Circle groups of 4. Then draw the triangles into two equal rows.<br>  <br>_____<br>_____<br><br>Use the array of smiley faces to answer the questions below.<br>a. _____ rows of _____ = _____<br>b. _____ columns of _____ = _____<br>c. _____ + _____ + _____ + _____ = _____<br>  </td></tr>
<tr><td>Topic C: Rectangular Arrays</td><td>  <br>  <br>Write a repeated addition sentence for the array.<br>_____</td></tr>
<tr><td>Topic D: Even and Odd numbers</td><td>Use the objects to create an array.<br><table><tr><td>a.<br>  </td><td>Array<br><br>There is an even/odd (circle one) number of circles.</td><td>Redraw your picture with 1 less circle.<br><br>There is an even/odd (circle one) number of circles.</td></tr></table></td></tr>
</table>

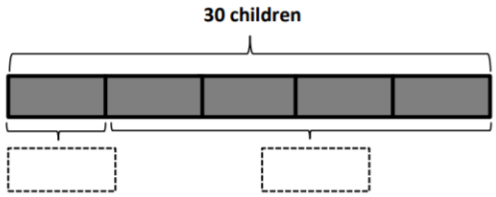
In grade 3, students build upon the knowledge they gained in 2nd grade and are introduced to

the idea of multiplication as a new operation, “factors” and the “times” (x) symbol in the progression below:



Students’ understanding of multiplication is scaffolded through the support of pictures, sentence stems, and manipulatives. Below are examples of student work from each topic:

| | |
|---|---|
| **Topic A: Multiplication and The Meaning of Factors** |  c. 6 + 6 + 6 = _____ _____ groups of six = _____ 3 × _____ = _____ |
| **Topic B: Division as a factor of an unknown problem** | There are 8 birds at the pet store. 2 birds are in each cage. Circle to show how many cages there are.  8 ÷ 2 = _____ There are _____ cages of birds. |
| **Topic C: Analyze Arrays to Multiply units of 2 and 3** | 6 × 3 = _____  (4 × 3) = _____ (2 × 3) = _____ (4 × 3) + (2 × 3) = _____ + _____ 6 × 3 = _____ + _____ _____ × 3 = _____ |

| | |
|---|---|
| **Topic D: Division using units of 2 and 3** | How many students are sitting at each table?   |

**Kindergarten**

**Common Core Progression:**

1. Students learn to quickly recognize how many is in a small group without having to count the objects (perceptual subitizing)
2. Next, kindergarten students develop conceptual subitizing where they recognize that a group of objects is composed of two subgroups and quickly combine the two subgroups to find the cardinality of the collection. Example: seeing a set as two subsets of 2 and saying "4." Conceptual subitizing when adding and subtracting small numbers progresses and supports more advanced methods for adding, subtracting, multiplying and dividing in later grades.
3. Students act out adding and subtracting situations by representing quantities in the situations with objects, their fingers, and math drawings.

**Sansu Math Curriculum:** In the first unit of the first grade Sansu Math, students become interested in numbers and quantities by having them create groups of objects based on specific characteristics they choose, and comparing the size of groups. Students think about how to represent group sizes using words and semi-concrete representations, and how to compare group size using one-to-one correspondence. They should understand how to count up to and including 10 objects and how to read and write numerals up to and including 10.

## 11. Unit Plan: Multiplication

| Lesson | Learning goal | Tasks |
|---|---|---|
| 1* Research Lesson | Recognize situations in which items are organized in equal groups and identify the number in each group and the number of groups. | Students investigate the number of students on amusement park rides and notice the difference between equal and unequal groups. |
| 2 | Understand that equal-groups situations can be represented by multiplication math sentences by using "number in each group," "number of groups" and "total number (product)." Represent given situations using multiplication math sentences. | Students will be using the multiplication symbol for the first time. Students will understand that the first digit in the number sentences is the number of objects in each group followed by the number of groups. |

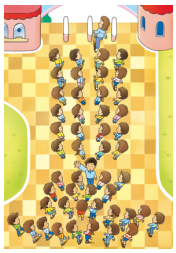
| | | |
|---|---|---|
| 3 | Solidify understanding of multiplication by representing multiplication situations using math sentences and counters. | Students will write multiplication sentences using "x" in order to describe pictures of objects in equal groups. |
| 4 | Understand that the total (product) can be obtained by adding the multiplicand to itself the number of times as the multiplier show. | Students view a picture without all of the items shown. They will use a multiplication sentence to represent the problem and an addition sentence to solve. |
| 5 | Understand even and odd numbers. | Students determine whether a number is "even" or "odd" by viewing it in groups of 2. |
| 6 | Learn the meaning of "times as long" and understand that multiplication can be used to find the length which is "times as long as" a certain length. | Students will learn that multiplication can be also use to represent length situations using a tape strip. |
| 7 | Strengthen understanding of multiplication and see it as useful for concisely describing the world around them. | Students will find and draw equal group situations at school. Students will write multiplication sentences to describe the situation. |

**7. About the Unit and Lesson**

This is the first time students encounter multiplication; they have experienced foundational ideas earlier in kindergarten, first grade, and second grade (such as skip-counting by 2s, 5s, 10s, and 100s). Although the Common Core would suggest that students are introduced to multiplication in third grade, we have talked to third grade teachers and read the Primary Math curriculum, and decided that laying a strong foundation in multiplication will benefit students. The unit is designed to give students numerous opportunities to notice and discuss equal-group situations. Students can see the situations and recognize that we can multiply the number of objects in each group by the number of groups to determine the total number of objects. Students will work independently but they will also spend time sharing their thinking and defending their mathematical arguments during a daily whole group math discussion. Students will take part in critiquing their own work and the work of their peers, and relating multiplication to the equal-group situations. Through these problem-solving experiences we hope that students will gain academic success and also develop the skills to discuss and defend mathematical arguments, as well as notice and wonder about mathematical situations. Students at Chavez are majority English Language Learners and therefore the

opportunity to not only do math but talk about math is extremely important to their ability to problem-solve and make sense of mathematical concepts.

## 8. Lesson Plan

| Steps, Learning Activities<br>Teacher's Questions and Expected Student Reactions | Teacher Support | Points of Evaluation |
|---|---|---|
| **1. Introduction**<br>  <br>There are 2 classes going to the 47th Street Carnival. "Count the number of students going into the carnival. How many students are lined up neatly?"<br>"T+T how you counted"<br>**Anticipated Responses:**<br>- I counted by 1s.<br>- I made 2 groups of 10 and then had 4 left over.<br>- I counted by 2s.<br>- I counted 4 across and then the next four across until I got to 24.<br>- I counted 6 down and then another 6… until I got to 24.<br><br>"How many students are lined up at the bottom?" (only give students a minute to count)<br>**Anticipated Responses:**<br>- 22-25<br><br>"Why is it harder to count this group?"<br>**Anticipated Responses:**<br>- The students are all over.<br>- The students are not lined up neatly.<br>- The students are not in equal groups. | | Are students thinking about the number of children in each line?<br><br>Are students noticing that the bottom group of children is harder to count?<br><br>Are they arranging their counters in equal groups or in groups of 5 or 10? |

| | | |
|---|---|---|
| "There are the same number of kids in each class! How should the children line up so we can count more easily? Use the counters to show your idea."<br><br>Students use 24 counters.<br>**Anticipated Responses:**<br>- Counting by ones<br>- 2 groups of 10 and a group of 4<br>- 12 groups of 2<br>- 2 groups of 12<br>- 4 groups of 6<br>- 3 groups of 8<br>- 4 groups of 5 and a group of 4<br><br>"What did you notice when you arranged the counters?"<br>**Anticipated Responses:**<br>- There is more than one way they can line up nicely.<br>- Putting the students in equal groups makes it easier to count. | | |
| **2. Noticing and Wondering**<br><br>Show the amusement park picture on page 166.<br><br>"What do you notice and wonder?"<br><br>**Anticipated Responses:**<br><br>- It's an amusement park, I see a train, teacup, and boats, there are 12 kids riding in the boats…<br>- What do you notice about the number of children on each ride?<br><br>**Anticipated Responses:**<br><br>- There are 4 kids on each boat<br>- There are 3 kids in each teacup<br>- There are 2 kids in each go-cart | Have students revoice each other<br><br>Highlight an interesting or important noticing or wondering | |

| | | |
|---|---|---|
| **3. Posing the Task**<br><br>Let's investigate how many children are on each ride.<br><br>Under "My Idea", write down what you notice about the number of children on each ride. | | |
| **4. Anticipated Student Responses**<br><br>*Ideal Responses:*<br><br>- "There are 5 children on each train car and there are 2 train cars".<br>- "There are 4 children on each boat and there are 3 boats."<br>- "There are 3 children on each teacup and there are 4 teacups."<br><br>*Other Responses:*<br><br>- "There are 12 on the teacups."<br>- "There are 10 kids on the train".<br>- There are 60 kids at the amusement park."<br>- "There are 3 kids in some of the ferris wheel but not all of them." | How did you count the number of children on _____?<br><br>How many students are in each _____? How many_____ are there? | Are students noticing the number of groups and how many children are in each group?<br><br>Are students counting by ones? |
| **5. Comparing and Discussing**<br><br>Discuss the teacups, trains, boats, and cars.<br><br>Ask, "What do these rides have in common?"<br><br>**Anticipated Response:**<br><br>- They all have have the same number of children in each of the groups.<br><br>Emphasize equal group situations<br><br>"Which ride is different from the others? Why?" | Turn and Talk<br><br>Have students revoice their peers<br><br>Encourage student questioning<br><br>Have students refer to the amusement picture | Do students see the similarities in the way children are sitting on the teacups, trains, boats, and cars?<br><br>Do students notice what is different about the children on the ferris |

| | | |
|---|---|---|
| **Anticipated Responses:**<br><br>- The ferris wheel is different from the others.<br>- All of the other groups have equal groups. The ferris wheel has groups of 2, 3, and 4 children.<br>- It's harder to count the number of kids on the ferris wheel.<br><br>Discuss the uniqueness of equal groups situations. | | wheel?<br><br>Do students see the benefit or uniqueness of equal group situations? |
| **6. Summing up**<br><br>**Reflection:**<br><br>**Reflect on the lesson.** What did you notice? What do you wonder? What did you learn?<br><br>(Emphasize reflections that notice the importance or uniqueness of equal group situations). | What was interesting about the number of children on each ride?<br><br>How did you count the number of children on each ride?<br><br>What was different about the children on the ferris wheel? | |
| **7. Reflection**<br><br>Today I learned…<br>I notice that…<br>My friend _____ taught me that…<br>I made a mistake because…<br>Next time I will...<br>I wonder... | | What are students writing as their new learning?<br><br>What do students identify as a mistake or new learning? |

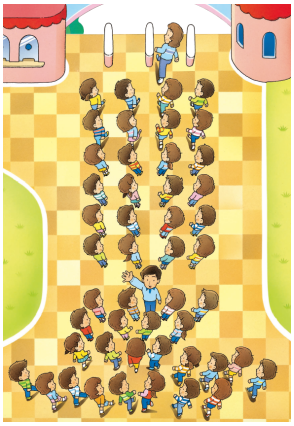
**9. Evaluation**

For this lesson, we will use students' reflections to evaluate what students learned about equal group situations. We will also use student work and teacher observations to answer the following questions:

- Related to research theme: Are students using mathematical language and models to describe and justify their thinking, as well as respond to, add on to, and critique the strategies of their peers?
- Related to lesson and unit goals:

- Do students recognize when objects are organized in equal and unequal groups?
- Can students will explain what makes equal groups different from unequal groups?
- Are students able to explain the usefulness of equal group situations to count more easily?

## 10. Board Plan



Jaime:
Circles in 4 groups of 6

Valeria C:
Circles in 3 groups of 8

Israel:
Circles in 6 groups of 4


Student 1: There are 3 children in each of the 4 teacups

Student 2: There are not the same number of children in each ferris wheel car.

Reflection

Noticings:
Wonderings:

## 11. Post-lesson Reflection