# 5th grade - Dividing a Fraction by a Whole Number

## Team Members

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| Lesson Date: | Instructor: | Grade Level: |
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| *5/16/2019* | *Alex Johansen Laughlin* | *5* |

## 1. Title of Lesson

*How do mathematician's divide a fraction by a whole number?*

$\frac{1}{2} \div 5$

## 2. Research Theme

Teach scholars to make sense of problems and persevere in solving them by teaching math through problem solving.

Teach scholars to construct viable arguments and critique the reasoning of others through note-taking, board work, and students' discourse.

## 3. Background and Research on the Content

Up to this point, students have worked with the division of whole numbers (2-digit/2-digit and 3-digit/2-digit) and with the division of decimal numbers by whole numbers. They also have background in multiplication of a fraction by a whole number and a fraction by a fraction.

Analysis of historical student performance data has shown that fraction division is one of the most difficult concepts for our fifth grade students. We chose to focus on this topic for this reason, and because we wanted to deeply investigate how students' conceptual understanding of division of fractions in fifth grade would affect their understanding of sixth grade mathematics (multiplying by the reciprocal as related to finding a fraction of a fraction/division with fractions). In addition, we wanted to observe what concepts students drew from to concretize their understanding of dividing a fraction. Will they use whole number division strategies? Will they apply ideas from multiplication of a fraction by whole number? Will they use equivalent fractions to create more units?

Related to students' difficulty with fractional division, we have found that our fifth graders often confuse the concepts of dividing a fraction by a whole number with dividing a whole number by a fraction. In order to help students to better conceptualize the difference, we propose to change the order of teaching these skills from what the lesson sequence in our curriculum currently suggests. In previous years, our lesson sequence had students dividing a whole number into fractions in quotative division contexts (e.g. *8 pieces of paper are used to make signs that are each ½ a sheet, how many signs can be made?*) before having them divide a fraction into smaller pieces (*1/2 a liter of water shared among eight people*). In the past, we have found that students become confused with the meaning of each. Here, we propose that we reverse the order of fractional division by beginning the division bend of this unit with dividing a fraction by a whole number.

We know that students benefit from an early understanding of the differences between models and the differences between types of division. Because of the aforementioned confusion, this research lesson seeks to find out if changing the order of the teaching of fractional division will affect how students differentiate between dividing a fraction by a whole number and dividing a whole number by a fraction. This can only be determined once we have finished teaching the unit. For the purposes of this research lesson, though, the two most permeating mathematical concepts that we want to address involve the following ideas:
- when we don't have "enough" of something (for whatever the mathematical situation is asking), we can decompose the original unit to make more, smaller units
- when we can't use the numbers and/or units that are given to us in a problem, we can substitute them with different units of the same value

We want to learn about the extent to which students have internalized these basic mathematical ideas, can apply tools they have used in different contexts (equivalent

fractions), and can articulate how using these tools helps them solve problems involving division of fractions.

## 4. Rationale for the Design of Instruction

The team used Eureka Math grade 5 module 4 to begin thinking about how to best design a unit that helped students use what they know about multiplying and dividing whole numbers to make sense of multiplying and dividing with fractions. In this unit, students learn to multiply fractions and decimal fractions and begin working with fraction division. Students begin to think about fractions of a group and learn that the "whole" is not always one. Students learn that we can take a fraction of a set (e.g. ¾ of 12 = ¾ x 12) and that we can take a fraction of a fraction (e.g. 1/3 of ½ = ⅓ x 1/2). Next, students focus on interpreting fractions as division. Equal sharing with area models provides students with an opportunity to understand division of whole numbers with answers in the form of fractions or mixed numbers. They learn to multiply mixed number by mixed numbers, and multiply fractions by decimals. Tape diagrams provide a linear model of these problems while many students prefer to use area models to prove their thinking.

The division bend of this unit begins with students quickly dividing various units into groups that are easily divisible because the dividend is a multiple of the divisor (e.g. 2 pencils ÷ 2 = 1 pencil; 8 ninths ÷ 4 = 2 ninths). We expect that students will see that the case of $\frac{1}{2} \div 5$ is not like the previous day's problems because the numerator of the dividend is not a multiple of the divisor. Students learned the meaning of equivalent fractions in fourth grade (and have used it as a tool to create common units for adding, subtracting, and comparing fractions in fifth grade) and we want to see if students will once again reach to equivalent fractions as a tool to solve a problem. The difference with using equivalent fractions in this division context that instead of using equivalent fractions to make the denominators (or numerators) the same, here, they will be using this strategy in a novel way - to create more units.
Students use tape diagrams and area models to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number (5.NF.7). Using the same thinking developed previously to divide whole numbers, students can reason about being able to divide something when there are enough of the unit to equally share.

As for the context of this problem, we originally thought to use liters, because of its ability to be converted into various units when tenths are the denominator. However, when thinking about having students model the problem, we thought about the fact that water cannot really be "cut". Because our lesson requires students to think about decomposing a unit to make more, smaller units, cutting pieces of a brownie pan make more intuitive sense than cutting water. Also, the brownie pan works well because there are not units within the units, as there would be in a situation of a pan of rice (rice grains). Further, the brownie pan allows us to keep the focus on the rectangular pieces easily being decomposed, as opposed to confusion that might come with the depiction of a context like a cake pan, which implies a circular shape.

## 5. Goals of the Unit

Students will use what they know about division of whole numbers and fraction multiplication to make sense of dividing fractions by whole numbers and dividing fractions by whole numbers. They will justify their ideas using models and use equivalent fractions to problem solve about dividing fractions by whole numbers and dividing whole numbers by fractions.

## 6. Unit Plan

| Lesson | Learning goal(s) and tasks |
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| **1** | ***Mini Problem Solving Day***<br><br>***Lesson Goal:*** *Students will understand that when we divide a unit fraction by a whole number (when the numerator of the dividend is a multiple of the divisor), we can divide in the same way we divide whole numbers.*<br><br>***Task:*** *There is 8/9 of a meter of ribbon that is going to be equally distributed among four friends. How much of the whole meter is each friend going to get?*<br><br>*Expression: 8/9 ÷ 4*<br><br>***Practice:*** *Problems dividing various units by divisors that are multiples of the dividend.* |

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| **2**<br>Research Lesson | ***Problem Solving Day***<br>Adapted from Eureka Math Lesson 26<br><br>***Lesson Goal:*** *Students will understand that when we divide a unit fraction by a whole number, we need to create an equivalent fraction that can be divided by the divisor.*<br><br>***Task:*** *Emilio has half a pan of brownies. He wants to equally distribute the half pan to his five friends. How much of the whole pan will each friend get?*<br><br>Expression: ½ ÷ 5 |
| **3** | ***Mini Problem Solving Day***<br><br>***Lesson Goal:*** *Students will understand that in order to divide a non-unit fraction by a whole number, we can apply the same strategy we used with dividing unit fractions, i.e. create equivalent fractions.*<br><br>***Task:*** *Ms. Laughlin is going to equally distribute ⅔ L of saline solution to 4 science groups. How much saline solution is each group going to get?*<br><br>***Practice:*** *Problems involving dividing fractions by whole numbers where the numerator of the divisor can easily be changed to a multiple of the divisor using equivalent fraction.* |
| **4** | ***Problem Solving Day***<br>Adapted from Eureka Math Lesson 25<br><br>***Lesson Goal:*** *Students will understand that when we divide a whole number by a unit fraction, we can think of it as how many groups of ____ fit into the whole number.*<br><br>***Task:*** *Nataly has six sheets of paper. She is going to pass these papers out to her friends to make signs for Special Olympics. Each sign will be on ½ of a sheet of paper. How many students got to make signs?*<br><br>Expression: 6 ÷ ½ |
| **5** | ***Mini Problem Solving Day***<br><br>***Lesson Goal:*** *Students will understand that when we divide a whole number by any proper fraction, we can still think of it as how many groups of ____ fit into the whole number.* |

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| | ***Task:*** *8 meters of ribbon are available to be used for a group project. Each project requires ⅔ meter. How many projects can be made with the 8 meters of ribbon?*<br><br>Expression: $8 \div \frac{2}{3}$<br><br>***Practice:*** *Problems involving dividing whole numbers by improper fractions where the whole number is a multiple of the numerator of the divisor.* |
| **6** | ***Practice Day***<br><br>***Lesson Goal:*** *Distinguish between scenarios that require division of a fraction by a whole number vs. a whole number divided by a fraction. Use reasoning to determine if quotient makes sense.*<br>Adapted from Eureka Math Lesson 27<br><br>***Practice:*** Problems involving both dividing a fraction by a whole number and dividing a whole number by a fraction. |
| **7** | ***Problem Solving***<br><br>***Lesson Goal:*** *Connect dividing by fractions to dividing by decimals (tenths, hundredths)*<br><br>***Practice:*** Problems involving converting decimals to fractions as a strategy to divide decimals. |

## 7. Relationship of the Unit to the Standards

| **Prior learning standards that unit builds on** | **Learning standards for this unit** | **Later standards for which this unit is a foundation** |
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| Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use | Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<br><br>Interpret division of a unit fraction by a non-zero whole number, and | Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models |

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| this principle to recognize and generate equivalent fractions.<br><br>Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.<br><br>Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* | compute such quotients.<br><br>Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*<br><br>Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *e.g., by using visual fraction models and equations to represent the problem.*<br><br>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | and equations to represent the problem. |

## 8. Goals of the Research Lesson

*Students will understand that in order to divide a fraction by a whole number, they must decompose all parts of the whole in order to know the unit fraction.*

*Students will understand that they can apply the strategy of creating equivalent fractions in order to create a dividend (with a numerator) that is more easily divisible by the divisor.*

## 9. Research Lesson

| Learning task and activities, anticipated student responses, key questions or comparisons that will build insights | Teacher support | Assessment (Points to Notice) |
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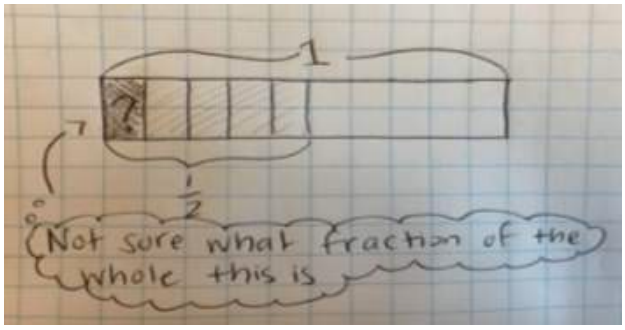
| ***Introduction***<br>*Refer to previous day's learning* | *T: What was our summary from yesterday?* | |
| ***Posing the Task***<br>*T poses question stem.*<br><br>*T: (writes and says)*<br>*Emilio has half a pan of brownies. He is going to equally distribute the half pan to his five friends. How much of the whole pan will each friend get?*<br><br>*T: What is the expression to represent the story problem?*<br><br>*Establish the expression as ½ ÷ 5*<br><br>Guiding Question: How do mathematicians divide a unit fraction by a whole number? | *T: What questions might I ask from this information?*<br><br>*S: How much of the whole pan will each friend get? How much is leftover?* | *Do students understand this as a division context? Do they identify equal sharing as division?*<br><br>*Do the students' expressions represent the story problem?*<br><br>*Do any students think of the problem as ⅕ of ½?* |
| ***Anticipated Student Responses***<br><br>1. S can draw the diagram, but doesn't know what to call the unit<br>  <br> | *T circulates to notice common misconceptions, anticipated responses, and novel strategies* | *Do students use previous days' work to solve?*<br><br>*What strategies do students use first?*<br><br>*Do any students use the inverse operation to solve?*<br><br>*Do struggling students guess and check?*<br><br>*If students use more than one strategy, which ones do they use?* |

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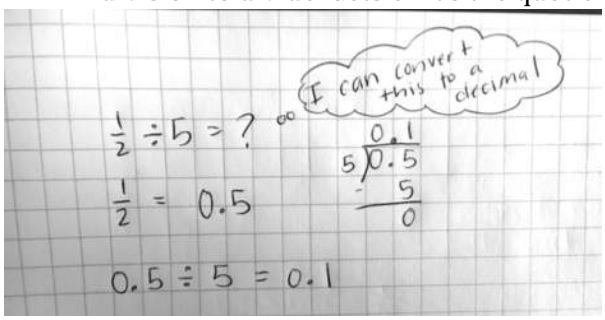
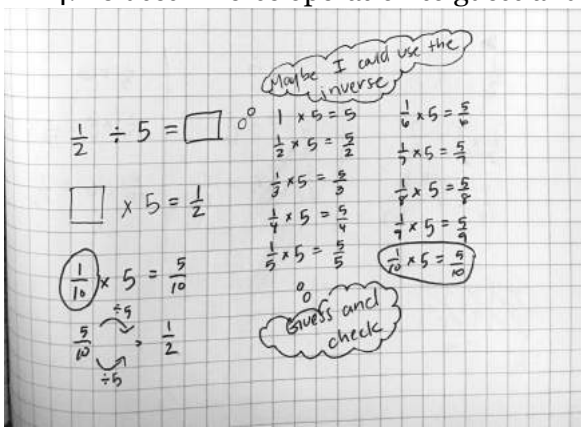
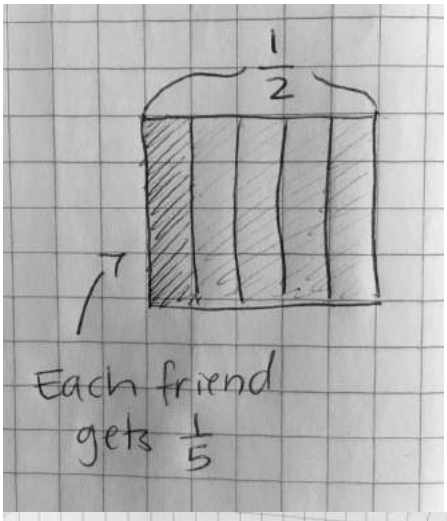
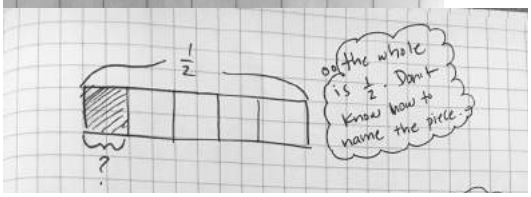
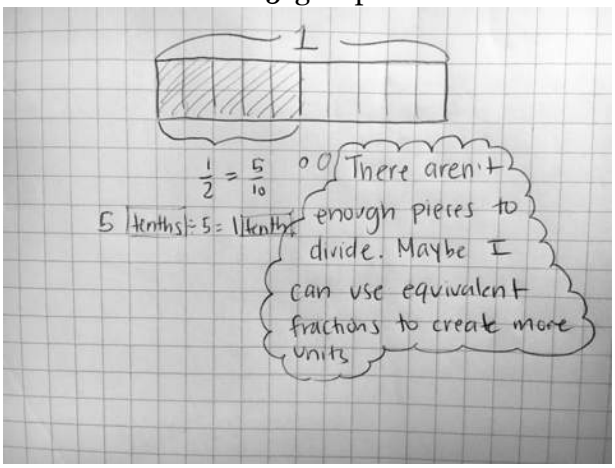
| 2. $\frac{1}{2} \div 5 = 10$<br>Student counts 10 pieces in the whole and incorrectly determines the quotient to be ten<br><br>3. Student converts the fraction to a decimal 0.5 and uses what he/she knows about decimal division to divide. Gets 0.1 as the quotient.<br>  <br><br>4. S uses inverse operation to guess and check<br>  <br><br>5. S makes ½ the whole and gets stuck. May incorrect identify the quotient as one fifth. | | *How do students explain their ideas in words?* |



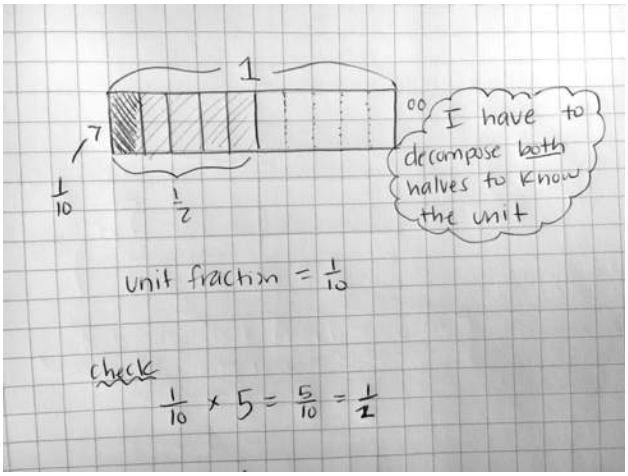



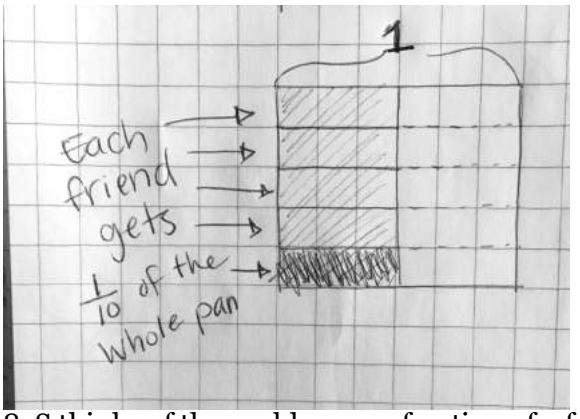

6. S changes the unit to tenths so that they can distribute to 5 groups



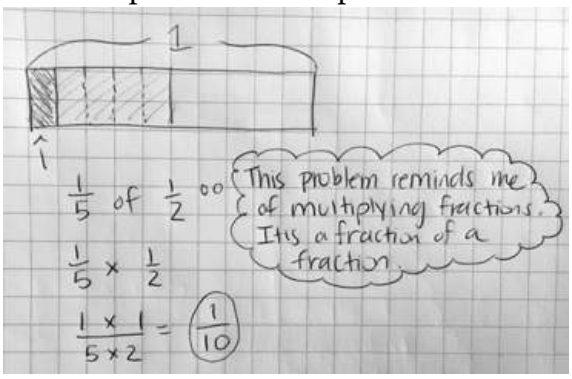

7. Student correctly understands that the quotient will be smaller than the divisor. Student decomposes both halves in order to figure out the new unit fraction.

$$\frac{1}{2} \div 5 = \frac{1}{10}$$




8. S thinks of the problem as a fraction of a fraction and multiplies to find the product



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| ***Comparing and Discussing, including Teacher Key Questions***<br><br>S1: Student incorrectly writes ½ as the whole, says answer is ⅕.<br><br>S2: Student decomposes tape diagram on both halves to make new unit of tenths<br><br>S3: Student realizes that we can't divide it because there aren't enough units.<br><br>Needs to change ½ into 5/10 so that it can be divided.<br><br>5 tenths ÷5 = 1 tenth<br><br>Connecting strategies:<br><br>- How are friend 2's strategy and friend 3's strategy related?<br>- Where do you see equivalent fractions in both of their ideas?<br>- Why did friend 2 need to decompose the other half?<br><br>- How was today's problem different from yesterday's problem?<br>- How did you solve the issue of not having enough units to equally distribute?<br>- Why did friend 3 create 5/10 as their equivalent fraction and not, for example, 2/4? | | *Can students prove their quotient with the inverse operation?*<br><br>*Can students prove with a model?*<br><br>*Do any students notice that this problem is asking for ⅕ of ½?*<br><br>*Do students go back to their original idea to revise/update their work?* |
| ***Summing Up***<br><br>*Summary: Today, as a hardworking class, we learned that we can divide a (unit) fraction by a whole number by:*<br>- *using equivalent fractions to create a dividend that is a multiple of the divisor.* | *Have students turn and talk about connections between strategies and the new learning for today.* | *Are Ss able to articulate that when we divide a fraction by a whole number, we are making the piece smaller?* |

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| *Extension:*<br>• ⅓ ÷ 2<br>• ¼ ÷ 3 | | |

## 10. Points of Evaluation

How does the relationship between teacher boardwork and scholar note-taking facilitate student creation of viable arguments and critiquing the reasoning of others?

Are scholars able to use mathematics to justify their arguments?

Are scholars able to use what they know about equivalent fractions to make sense of dividing ½ by 5?

Does the sequence of student ideas shared in the comparison and discussion serve to make the importance of equivalent fractions clear to students?

## 11. Board Plan