Lesson Research Proposal for 7th Grade Math

For the lesson on May 15, 2019 at the Chicago Lesson Study Conference

Mr. Bingea class, Brentano Math and Science Academy

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1. Title of the Lesson: Compound Probability

2. Brief Description of the Lesson:

In this lesson, students will first experience and compare two games that involve simple probability. Students will then be presented with two new types of games that involve compound probability. Students have never been asked to reason with compound probability and will be prompted through discussion to think about the total number of outcomes that a compound event produces and how it differs from a simple probability event. By giving students two games where the theoretical probability is not immediately obvious, we hope to create the need for students to make a sample space. Our aim is that students will be able to intuit a correct sample space after playing both games and explore different ways to keep track of all possible outcomes.

3. Research Theme:

The research theme has been SMP 3: Construct viable arguments and critique the reasoning of others.

Our hope is to have students of varying abilities and backgrounds respond to and build on the thinking of others in the class. In order to achieve this, students will analyze games in which they can either “win” or “lose” and be forced to decide which game has a more favorable probability for them. Using this structure, we believe students will be incentivized to articulate which game has more favorable outcomes, even with this being the first time they are exposed to compound probability.

4. Goals of the Unit:

At the end of the unit, students will understand and use the terms such as “event,” “sample space,” “outcome,” “probability,” etc. when analyzing probability contexts and questions. They apply their understanding by designing and using simulations to estimate probabilities of outcomes of chance experiments and understand the probability of an outcome as its long-run relative frequency. They will represent and understand sample spaces in various forms. They will use sample spaces to calculate theoretical probability. By the end of the unit, students will understand that the probability of any event falls on a scale that can be defined with words, fractions, and percentages. They will know that theoretical probability can be calculated in different ways and they will show an understanding by comparing theoretical and observed probability.

5. Goals of the Lesson:

At the end of the lesson, students will understand that compound probability consists of multiple, independent events. Therefore, there are more possible outcomes than each individual event. Furthermore, students will be able to generate a sample space and understand its need when determining the probability of compound events.

6. Relationship of the Unit to the Standards:

Related prior learning standards	Learning standards for this unit	Related later learning standards
CCSS.MATH.CONTENT.5.NBT.A.3 Read, write, and compare decimals to thousandths. CCSS.MATH.CONTENT.6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	CCSS.MATH.CONTENT.7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. CCSS.MATH.CONTENT.7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. CCSS.MATH.CONTENT.7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. CCSS.MATH.CONTENT.7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	Link to Highschool: Conditional Probability & the Rules of Probability

7. Background and Rationale: Bingea

Background:

This seventh grade class consists of 33 students with wide range of abilities and needs. It consists of students who take an enrichment Algebra course and also a group of students who have had exposure to probability through an after-school math enrichment program last year. At the same, this class consists of students who receive math intervention services. It is a co-taught class--with general education teacher and special education teacher--that contains seven students with IEP's and five English Language Learners, as well as two students who receive support from a Special Education Classroom assistant to help them access the lesson. Because of this diversity in student need, we have grouped students in heterogeneous groups to problem solve and learn from each others' ideas and mathematical strategies. To eliminate helplessness and the feeling of frustration we have attempted to design all lessons with multiple points of entry so that all students have access to the grade-level learning targets. All problem solving tasks start are presented first with a picture so that all learners can engage and make meaning of the problem context.

Rationale for learning goal:

Our team decided to focus on the introduction of compound probability, because we felt that it was the most difficult for 7th grade students to reason with. In past treatments of this unit, we have had to explicitly give students the strategies for determining compound probability by showing them step-by-step how to find the sample space with organized lists and compound probability. As a result we feel that students understand compound probability at a procedural level and lack the conceptual understanding of the impact of compound events on theoretical probability. In our research lesson we aim to have students intuit the difference between compound and simple probability and generate strategies to keep track of the sample space for a compound events.

Rationale for lesson design:

We have designed the lesson to initially draw upon students' previously learned understanding of simple probability to compare two games involving 1 event. Here we want to give all students an opportunity to justify why one game is more probable, bringing out the idea that we can assign a numeric probability by knowing the total number of possible outcomes and the number of favorable outcomes. We also want to bring out the idea of a sample space as a way to justify and visualize theoretical probability. For engagement purposes and to familiarize students with the problem context, we decided to have students play the two games before comparing. After we play the two games, students results from this first task will stay posted on the board so they can compare and contrast the single probability event with the later compound probability games. Additionally, we want the students to write out the observed outcomes as a way to organize their thought process around the outcomes of the game.

8. Research and *Kyozaikenkyu:*

In order to understand how probability is taught the team researched various curricula to provide background and rationale as to how this lesson should be taught. Up until seventh grade, students have no formal instruction on probability but rather have been exposed to it in everyday life using language of things being likely, unlikely or impossible. In 7th grade, the common core formally introduces students to the topic of probability. Here students begin to grapple with the concepts of the chance of events happening both with simple events and compounds event. Students learn to record the possible outcomes using organized lists, tree diagrams and tables. At the same time, they express these events in the form of a fraction, decimal and percentage using the scale of zero to one.

The two curricula the team researched were Engage NY and Illustrative Mathematics. The team also consulted John A. Van De Walle *Elementary and Middle School Mathematics Teaching Developmentally*. Both Engage NY and Illustrative Mathematics overall follow a similar sequence beginning with lessons on simple probability and moving to compound probability, each emphasizing the need for students to express the outcomes in the form of a fraction, decimal and percentage. Engage NY, however, places a greater emphasis on theoretical probability rather than experimental probability. They introduce students to a number line model of zero to one, to help them visually see where the probability would fall between impossible and certain. In addition, Engage NY immediately introduces procedural notations of probability within the first four lessons before students have even had a chance to grapple with the idea of what could theoretically happen versus what actually happens.

In contrast, Illustrative Mathematics takes a slightly different approach. Students are not rushed into the procedural notations of probability but rather get a chance to observe real life outcomes by playing various games that involve spinners, coins and dice. From these experiences, students develop an understanding of the relationship between theoretical and observable outcomes.

According to Van De Walle, the problem with how probability is introduced is students do not get enough opportunities to play games. He argues that without students playing games and seeing the observable outcomes many times, students cannot develop a conceptual understanding of what probability is and means in the real-world. Based on his analysis, the team decided to create the 7th grade unit plan and lesson on probability under the notion that students have a chance to play multiple games. When students experience probability by generating their own observable outcomes, they can create authentic reasoning and articulate their thoughts when responding to others.

9. Unit Plan

	Learning goal(s) and tasks	Vocabulary	CCSS
	Simple Probability		
1	Students understand that a probability is a number between 0 (impossible) and 1 (certain) that represents the likelihood that an event will occur. Students interpret a probability as the proportion of the time that an event occurs Task - place given scenarios on a likelihood scale (Impossible, unlikely, equally likely as not, likely, certain)	Impossible Equally Likely Certain Probability	CCSS.MATH.CONTENT.7.SP.C.5

2	Students learn that we can define probability in terms of a fraction, decimal, percents Task - ● In the first round, one of you will score on an even roll and one of you will score on an odd roll. You decide that first. ● In the second round, the winner of round one will score on numbers 1−4, and the other player will score on numbers 5−6. ● Each round is ten rolls. Be sure to turn on "History" after your first roll and wait for it to update before rolling again. When each player had three numbers, did one of them usually win? When one player had four numbers, did you expect them to usually win? Explain your reasoning.	Possible outcomes	CCSS.MATH.CONTENT.7.SP.C.5
3	Students can write out the sample space for a simple chance experiment. Task - Students took data on coin flips and rolling a die and used all possible outcomes to generate a sample space	Sample space	CCSS.MATH.CONTENT.7.SP.C.5
4	Students can explain whether certain results from repeated experiments would be surprising or not. I can estimate the probability of an event based on the results from repeating an experiment. Task - each student pulled a block from a bin (10 blocks - 5 blue, 4 green, 1 white). Class then discusses to determine whether or not outcomes were surprising.	Observed probability	CCSS.MATH.CONTENT.7.SP.C.6
5	Students can explain why results from repeating an experiment may not exactly match the expected probability for an event. Students can calculate the probability of an event when the outcomes in the sample space are not equally likely. Task - students reflect on the block activity from prior lesson to determine if observed outcomes are close to theoretical probability, which they will determine in groups.		CCSS.MATH.CONTENT.7.SP.C.7
6	Students will see the usefulness of comparing probability in terms of a fraction, decimal, and percent. Task- students will perform two experiments with similar theoretical probabilities. They will use sample spaces to		CCSS.MATH.CONTENT.7.SP.C.7

	determine and compare probabilities in terms of fractions, decimals, and percentages.		
	Compound Probability		
7	**Research Lesson**: When given a compound event, students will feel the need to create the sample space to find the probability.		CCSS.MATH.CONTENT.7.SP.C.8
8	Formalize strategies to find theoretical probability of compound events Task- Students are given a compound sample of a sandwich shop and are asked to generate what the two events are.	Tree diagram Organized list Table	CCSS.MATH.CONTENT.7.SP.C.8
9	Students will design and use a simulation to estimate probabilities of compound events Task - each student will design a multi-step task and set the boundaries for what constitutes “winning”. Students then make a poster and display their work for others to play. At the end, students compare theoretical and observed outcomes.		

10. Research lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction: Students will be introduced to the fact they will be split into two teams with each playing a game.		
Posing the Task 1: Team 1 you're going to play game A. Team 2 you're going to play game B. Now team one you are first up. We are going to take a volunteer. Let's have ________ come on up. We are going to play the game 10 times and see who wins. We are going to record all the outcomes we observe. Team 1: For Game A, you have a coin with heads and tails. You need to flip a heads to win. (Play game A 10 times with students recording responses) Team 2: For Game B, you have a die that you will roll. You need to roll BELOW a 3 to win. (Play game B 10 times with students recording responses) Looking at the results from game A and game B, what do you notice? What do you wonder? If we played again, which game would you choose? Discuss in your groups.	DL support - check for understanding with individual students - make sure they are recording with appropriate notation Record results on board to mirror what students record in notebooks. Record results on board to mirror what students record in notebooks Notice/wonder chart on board. DL support - verbally prompt students based on observed need Record students noticings and wonderings. DL support - verbal prompting as needed	Are students recording correct data? Are any students making fractions to represent outcomes? Are students comparing observed outcomes? Are students comparing theoretical outcomes? Are students grappling with the difference between observed and theoretical outcomes? What arguments are students relying on? Are these valid arguments?

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<td>Anticipated Student Responses:
Notice/Wonder Anticipated Responses:
● The games are not fair
● Game __ won/loss more
● Game __ has a probability of ___
● There are ____ number of possible outcomes for game ___
● Flipping a coin is easier to win than rolling a number cube
● Why did we have to play game ____?

Game A & Game B Anticipated Responses:
Misconceptions:
● The die is easier because there are two good/favorable outcomes - 5 AND 6 both work
● Thinking ⅓ is bigger (closer to certain) than ½
● Based on luck
● Picking fractions to represent the observable outcomes and using those to justify which game will win more.

Desired Responses:
● ½ is bigger than ⅓
● It’s closer to certain on the number line
● Voice understanding that the theoretical and observable outcomes can differ</td>
<td>Ask students who say desired response ½ and ⅓ to explain why. Students often using the format or “My claim is…, my warrant is... Record the outcomes on the board to address misconceptions.</td>
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<td>Posing the Task 2:
I have two more games for us to look at.

Team 1 you are going to play game C, you have to flip two coins simultaneously and you win if you get a heads.

Team 2 you are going to play game D. You have a spinner with digits 1-4 and a dice with digits 1-6. To win you need to simultaneously roll the die, spin the spinner, and get an odd sum.

Which game has a better chance of winning?</td>
<td>DL supports - check for understanding regarding</td>
<td>Do students view this as two separate events?</td>
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(Students write down which game they think will win) (Play game C 10 times) (Play game D 10 times) What is different about games C and D when we compare them to games A and B? If we played again, which game would you choose? Why? (Write in notebook with evidence) Take quick student responses. After quick responses, teacher pulls student argument for game C being a better game. Ideally the misconception game C has a probability of ½. Students respond to argument. Once the correct sample space has been provided and defended for game C, we will conclude the probability of flipping two heads is ¼. The class' attention will be turned to game D. Now that we have settled on game C being ¼, discuss at your table, what is the probability of game D? Pull two different sample spaces on the board and have a discussion about which one is correct.	question posed - verbally prompt to get started - scribe if needed Teacher records observed outcomes on board Teachers will circulate and find students who argue for both games DL support - verbally prompt students based on observed need Teacher notes responses and misconceptions on the board. Judgement not doled out Student arguments recorded on board. Student response of correct sample space written on the board. Teachers circulate and listen to student justifications	How do students notate the multiple events? Do students create a sample space?
Anticipated Student Responses: **Anticipated Responses Comparing A & B to C & D:** ● A & B have 1 and C & D have 2 ● There are more outcomes in C & D ● The chance becomes less **Anticipated Responses for Game C & D:** Misconceptions: ● Students ignore "sum of odd" and create fractional representations for	Scribe sample space and theoretical probability	

the spinner and die (spinner $\frac{1}{3}$ and die $\frac{1}{6}$). Therefore, students think $\frac{1}{2}$ is bigger than $\frac{1}{2}$, $\frac{1}{2}$ is bigger than $\frac{1}{2}$, so game C will win more • One team got lucky • Addition of fractions to determine probability (i.e. $\frac{1}{3} + \frac{1}{2} = \frac{1}{5}$) • Pick fractional representations for the odd numbers Students will say $\frac{1}{2}$ is the probability of Game C because you can get H or T • on each individual event instead of the sum (i.e. $\frac{2}{3}$ of the spinner and 3/6 of the die) • Students will look at the observed outcomes to justify fractional representations. (i.e. HH was 3/10 of the time so the probability is 3/10). • Confusion so no work produced Desired Responses: • Students correctly list possible outcomes and correct numeric representation.		
Summing Up: Now I have game E for game E you have a spinner with colors red, blue, yellow, and a die with digits 1-6. To win you need to simultaneously roll the dice, spin the spinner, and blue with an even number		Do students use sample space to list potential outcomes? Do students correctly list all possibilities? Do students compare possibilities?

11. Evaluation

- Are students making claims with warrants?
- Are students responding directly to the thinking of others?
- Are students creating sample spaces with appropriate notation?
    a. Listing outcomes using numbers/letters to represent each possibility
    b. Are students listing all possible outcomes?
- Do students display a shift in understanding between games A & B (simple probability and existing understanding of sample space) to C & D (compound probability and more possible outcomes in sample space)?
    a. Informal strategies to determine possible outcomes

12. Board Plan