Elementary School Teaching Guide for the Japanese Course of Study: Mathematics

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Japanese Ministry of Education, Culture and Sports

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Chapter 1. General Overview

1 History of the Revision

The 21st century is the so-called "era of the knowledge-based society" where new knowledge, information, and technology have become dramatically important in various aspects of life, such as politics, the economy, and culture.

This process of moving to a knowledge-based society and globalization will intensify international competition for ideas, knowledge, and human resources, and at the same time emphasize the importance of coexistence and global cooperation among different cultures and civilizations. In this atmosphere, it will become more and more important to nurture a "zest for living" that emphasizes scholastic ability, a rich heart and mind, and the harmony of a healthy body.

On the other hand, various studies such as the Programme for International Student Assessment (PISA) study done by the Organization for Economic Co-operation and Development (OECD) indicate the following about students in Japan:

- (1) difficulty in reading comprehension and with writing problems that require thinking, decision-making, and expressing;
- (2) problems in the widening distribution of scores in reading comprehension, that are reflections of problems at home where not enough time is spent studying, and there is a lack of motivation to learn, as well as a poor study and living environment;
- (3) lack of confidence in their ability; feelings of insecurity about the future; and lowered physical strength.

Consequently, to enrich the education for students who will live in the 21st century, in February 2005 the Ministry of Education, Culture, Sports, Science and Technology-Japan (MEXT) requested the Central Education Council to reform the standards for the curriculum, and also to improve the quality and skill of teachers and the infrastructure of education. In April of the same year, discussion on these subjects commenced. During this period, the Fundamental Law of Education and the School of Education Law were amended, and the balance between knowledge, morality, and health was reaffirmed (the Fundamental Law of Education, Article 2,1), as were the importance of fundamental and basic knowledge and skills, of ability to think, to make decisions, and to express, and of motivation for learning (School Education Law, Article 30,2). For school education, it is now stated in the law that it is necessary to promote these objectives in a harmonious way. Based on the amendment of the laws, these fundamental issues in education were discussed in the Central Education Council. The discussion lasted 2 years and 10 months, and in January 2008 the report, The Reform of the Curriculum for Kindergarten, Elementary Schools, Lower Secondary Schools, Upper Secondary Schools and Special-Needs Schools, was submitted.

Based on the problems our students are facing, this report gives the following points that were considered as the basis for reform as well as directions for each school level and subject:

- (1) Reform of the curriculum based on the amendment of the Fundamental Law of Education
- (2) Establishing a shared concept of "zest for life"
- (3) Acquiring fundamental and basic knowledge and skills
- (4) Fostering the ability to think, to make decisions, and to express
- (5) Allocating necessary hours of classes to acquire solid academic ability
- (6) Motivating students to learn and helping them develop sound study habits
- (7) Enriching teaching for fostering rich minds and good physical health.

Specifically regarding (1), in order to develop Japanese people who are courageous and rich in spirit and who can lead the world of the 21st century, the Fundamental Law of Education was amended for the first time in approximately 60 years, and new ideals for future education were established. In the School Education Law amended by the Fundamental Law of Education, the new objectives of compulsory education were ordained, and reform of the curriculum is required to be fully grounded in the revision of the objectives of each school level.

In (3), fundamental and basic skills such as reading, writing, and calculation are to be completely acquired, so that they can form the foundation for future learning as appropriate to each developmental stage. For example, in grades 1 through 4, the emphasis is given on understanding obtained through experience and repeated practice.

Building on this foundation, to foster the ability to think, to make decisions, and to express described in (4), learning activities, such as experiments, observations, and writing reports and essays, in which knowledge and skills are utilized are enhanced according to each developmental stage. In addition, in order to develop language ability to support these activities, it was pointed out that in each subject it is necessary to acquire basic Japanese language skills—reading out loud, reading silently, reading and writing kanji characters—followed by learning activities such as recording, summarizing, describing, and writing essays.

In order to foster a rich mind and good physical health (7), in addition to enriching character education and physical education classes, by emphasizing language skills in Japanese language class and other classes and by nurturing learning through experience, it was suggested that it is necessary to let students gain confidence in life by interacting with others, society, nature, and the environment.

Based on the report of 3/28/2008, the School Education Law Enforcement Regulation was amended, and educational guidelines for kindergarten and elementary and lower secondary schools were made public. Educational guidelines for elementary schools will be implemented ahead of schedule for courses such as mathematics and science from 4/1/2009, and will be fully implemented by 4/1/2011.

2 Basic Policies for the Revision in Mathematics

The revision of the curriculum for mathematics was based on the main reform policies in mathematics stated in the report of the Central Education Council.

In mathematics, considering its challenges, mathematical activities must be further enriched so that students will acquire fundamental and basic knowledge and skills, develop the ability to think and express mathematically, and increase their motivation to learn throughout elementary, lower secondary, and upper secondary schools, according to their developmental stage.

The first policy states a very important objective in mathematics education throughout elementary, lower secondary, and upper secondary schools. The recent Japanese study of the curriculum and the international study of academic competitiveness indicate no decline in Japanese students' technical skills such as calculation, but the students find difficulty in interpreting the meaning of a problem and they do not sufficiently use their acquired skills and knowledge in learning and daily life.

To correct these problems, the following concrete policy was adopted:

The fundamental and basic knowledge of numbers, quantities, and geometric figures are the foundation for daily living and learning. With the advancement of scientific technology, it is actively being discussed more than ever that mathematics and science education should meet international competitive levels. Consequently, to firmly establish the fundamental and basic knowledge and skills of numbers, quantities, and geometric figures in students, while retaining the importance of a systematic path, the content of mathematics may be partially overlapped, and the curriculum may adopt 'repeated learning' according to developmental stages and grade levels.

The second policy is about organizing the curriculum by incorporating repetitions of some topics according to the developmental stages and grade levels of students in order to firmly establish fundamental and basic knowledge and skills. It is a characteristic of mathematics that its contents are systematically organized and a continuous learning path is clearly defined. Considering this characteristic, this policy aims to develop a curriculum that provides better connections across grades within the same content domain, and gradually increases the levels of complexity.

Mathematical thinking and expression play an important role in rational and logical thinking as well as in intellectual communication. For this reason, instructional content and activities that foster mathematical thinking and expression should be clearly indicated. We will enrich the kind of teaching where students are taught to think systematically, in logical steps, by reasoning, and to understand the connections among words, numbers, mathematical expressions, figures, tables, and graphs. This kind of teaching will also allow students to learn appropriate usage, problem-solving, how to explain one's ideas clearly, and how to express and communicate one's ideas to others.

The third policy is about fostering mathematical thinking and exposition. In mathematics, as in spoken language, there are many ways to express ideas, using numbers, mathematical expressions, figures, tables, or graphs. It is important to enrich

learning activities where these methods are used to teach how to think, explain, and express one's ideas.

It is important that we motivate students to learn mathematics , and students should experience the meaning of learning and utility of what was learned. For this reason, the following objectives are emphasized:

- To help students understand numbers, quantities, and geometric figures through learning activities which serve as a basis for understanding their meanings.
- To help students feel progress in learning, such as a depth and broadening of understanding through repeated learning that is designed according to the developmental stage and grade level of each student.
- To help students apply what has been learned to activities in daily life, to the study of other subjects, and to learning more advanced mathematics.

The fourth policy is about increasing the motivation for learning. The objectives that are emphasized are as follows: to enable students to develop an understanding of number and geometric figures through experience with foundational learning activities at appropriate levels, to help them feel the progress in their learning by broadening and deepening their understanding via a curriculum that incorporates appropriate repetitions of topics, and to help them apply what they have learned and acquired to activities in their daily life, in the study of other subjects, or in learning more advanced mathematics.

Mathematical activities play an important role in helping students acquire fundamental and basic knowledge and skills, in increasing students' ability to think and express mathematically, and in enabling students to feel a joy and purpose in learning mathematics. To enrich the teaching of mathematics through mathematical activities—with experiential activities and an emphasis on language—concrete examples of mathematical activities should be provided in the curriculum for elementary and lower secondary schools; and in upper secondary schools, project based learning is introduced in the required subjects and in more popular elective courses.

The fifth policy is about enriching mathematical activities. The phrase "mathematical activity" was first introduced in the Course of Study (COS), 1988. Along with defining the meaning of mathematical activities, the revised COS also describes instructional activities in each grade.

In addition to the five policies described above, the report indicates the following for reforming mathematics in elementary schools.

In elementary schools, the emphasis is placed on enriching mathematical activities and building fundamental and basic knowledge and skills while fostering a rich sense for numbers, quantities, and geometric figures. It also raises the students ability to think and express mathematically by using what has already been learned in daily life. To achieve these goals, the following reforms will be made:

a. The structure of the domains are unchanged; they are "Numbers and Calculations," "Quantities and Measurements," "Geometric Figures," and "Quantitative Relations." To emphasize the importance of thinking and expressing mathematically—using words, mathematical expressions, tables, and graphs—the domain "Quantitative Relations" is introduced in lower grades.

- b. To enable students to acquire knowledge and skills regarding quantities and geometric figures, and to foster their ability to think and express mathematically, a part of the contents is overlapped between grades, emphasizing the importance of a systematic path of mathematics. This means contents are gradually developed, giving an opportunity to review what has been learned. Teaching by this repeated learning method is applied according to developmental stages and grades.
- c. In order to emphasize the importance of mathematical activities, the contents in each grade now has a section that describes these activities. In doing this, special consideration is given to the connection between elementary schools and lower secondary schools.

Concrete examples of mathematical activities are now included; for example, activities that promote the understanding of quantities and geometric figures using manipulatives; activities that require application of knowledge and skills in real-world situations; and activities in which students explore and explain methods of problem solving.

The report states the following about the reforms in each domain of mathematics while clarifying those that need to be emphasized:

- d. Under the domain "Numbers and Calculations," the report emphasizes understanding how to express whole numbers, decimal numbers, and fractions, and developing a rich sense for numbers. It also emphasizes:
 - Understanding the meaning of calculation;
 - Thinking about the method of calculation;
 - Acquiring and using calculation skills.

For example, in grades 1 and 2, in order to understand fractions, fundamental mathematical activities such as folding a paper in half are carried out. Through a curriculum that uses a repeated learning method, students are expected to completely acquire calculation skills of whole numbers in grades 1 through 4, and in grades 3 through 6 they should develop calculation skills of decimal numbers and fractions. In grades 3 and 4, estimations of results are taught, and students are expected to estimate the results of calculations and make appropriate reasoning.

- e. In the "Quantities and Measurement" domain, understanding various units of quantities and their measurements, enriching a sense for the size of quantities, and explaining how to find quantities such as area, are considered important. For example, in lower grades, direct comparison of quantities such as the length and area of actual objects is taught. In grades 3 through 6, students are taught to investigate and summarize conclusions about the relationship between units. In grades 5 and 6, using previously learned knowledge about area, calculations of the area of a rhombus and trapezoid are taught.
- f. In the "Geometric Figures" domain, understanding the meaning and properties of geometric figures, enriching a sense for geometric figures, and becoming able to use the skills to view geometric figures in learning and daily life are considered to be important.

For example, in all grade levels, students are asked to draw, construct, or tile with various geometric figures, and they are asked to compare their shapes and sizes. The geometric plane and solid figures are taught in a well-balanced manner. In grades 5 and 6, congruence, enlargement, and reduction of geometric figures are taught.

g. In the "Quantitative Relation" domain, expressing quantitative relationships with words, numbers, mathematical expressions, tables, and graphs, and raising the idea of a function as the change and correlation between two quantities are considered to be important.

For example, in grades 1 and 2, using simple tables and graphs, students are

taught to represent phenomena and situations they experience. In grades 3 through 6, students are taught to write mathematical expressions with \Box and with letters. In grades 5 and 6, the curriculum for teaching the concepts of proportion and inverse proportion are enriched.

3 Main Points of the Revision in Mathematics

(1) Objectives

The following describes the objectives of mathematics instruction.

Through mathematical activities students will: (1) acquire basic and fundamental knowledge and skills about numbers, quantities, and geometric figures; (2) cultivate their ability to consider phenomena from their daily lives with foresight to generate and organize logical thinking steps to follow through, and to represent those phenomena; (3) recognize the joy of mathematical activities and the merits of mathematical manipulation; and (4) foster a disposition to willingly make use of mathematics in daily life and studies.

(1) "through mathematical activities"

In this revision, the phrase "through mathematical activities" is used at the top of this section for the first time. The objectives to which the term "through mathematical activities" refers have the same structure as in the previous COS. "Mathematical activities" mean various activities where students willingly and purposefully work on mathematics. Mathematical activities can include many different activities. It is often thought that they are physical activities such as hands-on activities or experimentation, or activities that deal with concrete objects, but there are others. Although activities such as thinking about mathematical problems, exploring ways to extend and apply mathematical knowledge, and explaining and representing mathematical thinking may not involve concrete objects, they are considered as mathematical activities.

(2) "with foresight to generate and organize logical thinking steps by which to express those phenomena"

In this revision, the phrase "to express" was added. The ability to think and the ability to express are considered to be complementary. In the process of expressing their thoughts, students may realize their own good points or errors in their ideas. By expressing thoughts, they become better able to organize logical steps and produce better ideas. In class, they can express various ideas and they can learn from each other. For this reason, thinking and expressing are mentioned in parallel.

(3) "foster a willingness to make use of mathematics in daily life and in their studies"

In this revision, solidifying fundamental and basic knowledge and skills and using what has been acquired are emphasized. Mathematical knowledge can be used in many daily life situations and in one's studies. It is not limited to studying other subjects but it can be applied to the future study of mathematics. In this revision, the term "studies" was added, and the term "apply" was changed to "make a use of" to make these points.

(2) The Objectives and Content in Each Grade

1. Objectives and Structure of the Contents

In mathematics, the objectives and content are shown in each grade, and are described in the order below:

- (1) Objectives
- (2) Content

A. Numbers and CalculationsB. Quantities and MeasurementsC. Geometric FiguresD. Quantitative RelationsMathematical ActivitiesTerms and Symbols

(3) Remarks concerning content

The content taught in each grade is categorized into four domains: "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometric Figures," and "D. Quantitative Relations." This is to make it easy to see the complete content of the mathematics curricula, and to make its connections and range intelligible. In this revision, we include domain D, "Quantitative Relations" in grades 1 and 2. Due to this change, the contents in all six grades now consist of the same four domains. Furthermore, in this revision, following the four domains, "mathematical activities" has been added for each grade. For the configuration of curricular content for elementary schools, a summary is shown in Table 1, page 13, for each grade and each domain. The curricular content for lower secondary school is shown in Table 2, page 16.

2. Mathematical activities

As previously noted, "mathematical activities" mean various activities in which students willingly and purposefully work on mathematics. "Students engage willingly and purposefully" means students try to find new properties or new ways of thinking or try to solve a concrete problem. In order to help students experience the meaning of quantities and geometric figures, raise their ability to think, reason and express ideas, and let them feel the joy and meaning in learning mathematics through mathematical activities, it is necessary to teach them to have a sense of these objectives and to engage them in learning willingly. For this reason, simply listening to what teachers say and doing practice problems are not included in the definition of mathematical activities.

Mathematical activities can include various activities. They are often physical handson, experimental activities, but there are others. Thinking about mathematical problems, developing and applying one's knowledge, and presenting and explaining what they think are all activities that do not deal with concrete objects, but they are included in mathematical activities. In this revision, mathematical activities in the classroom are clearly characterized. And in order to enrich mathematical activities in classrooms, some example activities are listed in the contents of each grade.

In the beginning of the "Mathematical Activities" section in each grade, the following statement will be found:

(1) For the contents of "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometric Figures," and "D. Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.

This statement suggests that it is necessary to teach all the content in each curricular domain by conducting mathematical activities. However, that does not mean that we deny the appropriateness of teachers explaining something or assigning practice problems, as these are naturally necessary. The mathematical activities listed in the curricular content are considered typical activities students can do. There are so many different activities depending on content and learning progress, it is impossible to list all of them. As implied by the word "examples," these activities can be used directly, or one can use activities similar to those listed. Also, it is necessary for teachers to develop and use activities that are not provided here.

The summary of mathematical activities from grade 1 to grade 6 is shown below.

Grade 1

- a. counting concrete objects
- b. expressing the meaning and method of calculation
- c. comparing quantities
- d. identifying and making shapes
- e. expressing situations in mathematical expressions

Grade 2

- a. finding situations where integers are used
- b. finding rules in the multiplication (*ku-ku*) table
- c. estimating size
- d. drawing geometric figures; constructing them; tessellations
- e. drawing figures; writing mathematical expressions; explaining with them

Grade 3

- a. thinking of ways to calculate and explaining them
- b. comparing decimal numbers and fractions
- c. learning relations between units
- d. drawing geometric figures such as equilateral triangles
- e. sorting items and making tables

Grade 4

- a. estimating results of calculations and making judgments
- b. thinking about how areas can be calculated and explaining one's ideas
- c. measuring areas
- d. tessellating (tiling) with figures such as parallelograms and learning properties of figures
- e. investigating relationships among quantities in their surroundings

Grade 5

- a. thinking of ways to calculate and explaining them
- b. thinking about how areas can be calculated and explaining one's ideas
- c. constructing and drawing congruent figures
- d. explaining properties of geometric figures inductively and deductively
- e. choosing and using appropriate graphs and tables for different purposes

Grade 6

- a. thinking of ways to calculate and explaining them
- b. learning relations between units
- c. scaled drawings; finding symmetric figures
- d. solving problems by using proportions

3. Improvement of the content of each domain

The following is a summary of the main changes of contents in this revision enhancements, additions, or repositionings—organized by the domains.

A. Numbers and Calculations

The meaning, representation, and calculations of whole numbers are mainly taught as before, in grades 1 through 4. In this revision, in order to promote learning of fundamental and basic contents, a major emphasis is placed on overlapping some contents across grades. This allows curricula to devise more effective ways to connect contents within the same content strand across grades and to gradually raise the level of sophistication.

For example, in grade 1, "a simple representation of 3-digit numbers," and "addition and subtraction with simple 2-digit numbers" are introduced; in grade 2, "addition and subtraction with simple 3-digit numbers," and "multiplication between a simple 2-digit number and 1-digit number" are introduced; in grade 3, "simple division whose quotient becomes a 2-digit number" is introduced; and in grade 4, "the acquisition of calculation skills of whole numbers" is noted.

Instruction on the meaning and representation of decimal numbers and fractions is included mainly in grades 3 through 6. As with whole numbers, contents overlap across the grades. For example: in grade 2, simple fractions are introduced; in grade 3, "addition and subtraction with simple decimal numbers" and "addition and subtraction with simple fractions" are introduced; in grade 4, "multiplication and division with

simple decimal numbers" is introduced; in grade 5, "multiplication and division with simple fractions" is introduced; and in grade 6, "the acquisition of calculation skills of decimal numbers and fractions" is noted. As for the range of calculation with whole numbers, "addition and subtraction with 4-digit numbers" and "multiplication between 3-digit numbers and 2-digit numbers" are now newly noted. For decimal numbers and fractions, the *hadome* restriction^{*} is now removed.

The instructional content of "estimating the results of calculation" is included in grade 4, and stresses the students' ability to appropriately reason about ways of calculating and coming up with results based on estimates.

B. Quantities and Measurements

In addition to "length," "comparing areas and volumes" is noted in grade 1, and it is hoped that this will become the foundation for understanding units and measurements in grade 2 and beyond. The instructional content moved between grades includes, for example, "Units of volume (liter, etc.)" in grade 2; "Units of volume (cubic centimeters, etc.)" and "per-unit quantities" in grade 5; and "finding the area of a circle" in grade 6. The new instructional content in the six grade, "finding volumes of prisms and cylinders" and "understanding units in the metric system and how they work," is noted.

C. Geometric Figures

Plane figures and solid figures can be taught with sufficient balance in each grade. In grade 1, shapes that appear in the everyday life of students are used, and both plane figures and solid figures are taught. The instructional contents that have been moved between grades are: "square, rectangle, and right triangle" and "box shape" in grade 2; "isosceles triangle and equilateral triangle," "angle," "circle and sphere" in grade 3; "parallelogram, rhombus, and trapezoid" and "cube and rectangular prism" in grade 4; and "prisms and cylinders" in grade 5. The instructional contents that have been moved from lower secondary schools is "congruence of figures" in grade 5 and "scaled drawing" and "symmetric figures" in grade 6. These are noted to address the continuity of instructional contents between elementary and lower secondary schools. As new contents, "how to represent the location of a thing" in grade 4, and "polygon and regular polygon" in grade 5 are noted.

D. Quantitative Relations

In this revision, the instructional content for mathematical expressions includes: "mathematical expressions for addition/subtraction" in grade 1; "mathematical expressions representing the relationship between addition and subtraction" and "mathematical expressions for multiplication" in grade 2; "mathematical expressions for division" and "mathematical expressions with \Box " in grade 3; "mathematical expressions that use $\Box \Box$ and \triangle ," (partially moved from lower secondary school) in grade 4; and "mathematical expressions with letters" in grade 6. In regard to functions, "simple proportional relations" in grade 5 is noted, and connects to "direct proportion"

^{*} explicitly stating which items to teach and which not to teach

in grade 6 so that the level of complexity may be gradually increased. Also, in grade 6, "Inverse proportion" (partially moved from lower secondary schools) is noted.

As for organizing data, "representing quantities using pictures and diagrams" is noted in grade 1; "simple tables and graphs" is in grade 2; and "frequency distribution" and "possible cases" (partially moved from lower secondary schools) are noted in grade 6.

Table 1: Structure of the content of mathematics in elementary school

[] indicates new content; {} indicates overlapping contents between grades due to repeated learning
The <u>underline</u> indicates content moved between grades.

	A. Numbers and Calculations	B. Quantities and Measurements	C. Geometric Figures	D. Quantitative Relations	Mathematical activities
Grade 1	Meaning and Representation of whole numbers • 2-digit numbers and {simple 3- digit numbers} Addition and subtraction of 1-digit numbers {Addition and subtraction of simple 2-digit numbers}	Comparing quantities • comparing length, comparing [area and volume] <u>Telling time (moved from</u> <u>grade 2)</u>	 observation and structure of shapes encountered in everyday life ([plane figures], solid figures) 	Representation with mathematical expressions • expressing addition and subtraction situations (moved from A. Numbers and Calculations) [Expressing quantities using pictures and diagrams]	a. counting concrete objects b. expressing the meaning and method of calculation c. comparing quantities d. finding and constructing shapes e. expressing situations with mathematical expressions
Grade 2	Representation of numbers • 3-digit numbers, 4-digit numbers, {10000, simple fractions such as 1/2, 1/4}, etc. Multiplication of whole numbers • multiplication table (<i>ku-ku</i>) [up to 9×9] Multiplication of simple 2-digit numbers	Units and measurements of quantities • units of length (<i>mm</i> , <i>cm</i> , <i>m</i>) • <u>units of volume, (<i>ml</i>,<i>dl</i>,<i>l</i>) (moved from grade 3) Units of time: (days, hours, minutes), (moved from grade 3)</u>	 triangles, quadrilateral, <u>squares</u>, rectangles, right triangles (moved from grade 3) <u>shape of a box (moved</u> from grade 3) 	Representation with mathematical expressions • relationship between addition and subtraction (moved from A. Numbers and Calculations) • expressing multiplicative situations with mathematical expressions (moved from A. Numbers and Calculations) Simple tables and graphs (moved from A. Numbers and Calculations)	a. finding situations where whole numbers are used b. finding rules from a multiplication table (<i>ku-ku</i>) c. estimating quantities d. drawing figures, constructing figures, tessellations e. expressing ideas in diagrams and formulas and explaining them
Grade 3	Representation of whole numbers - 10000, {100000}, etc. Addition and subtractions with whole numbers • addition/subtraction with 3- digit and [4-digit numbers] numbers Multiplication with whole numbers • multiplication with 2-digit numbers and 3-digit numbers, (for example, 3-digit number × 2-digit number) Division with whole numbers	Various units and measurements • length (<i>km</i>) and weight (<i>g</i> , <i>kg</i> , [ton]) Measurement using tools Units of time: seconds, calculations with time	 isosceles triangle, equilateral triangle (moved from grade 4) angle (moved from) circle, sphere (moved from grade 4) 	Representation with mathematical expressions • <u>expressing situations of</u> <u>multiplication of</u> <u>division with</u> <u>mathematical</u> <u>expressions (moved</u> <u>from A. Numbers and</u> <u>Calculations)</u> • [relating mathematical expressions to diagrams; mathematical expressions that use □]	 a. thinking about ways to calculate and explaining them b. comparing sizes of decimal numbers and fractions c. learning relations among units d. drawing figures such as equilateral triangles e. sorting data and representing data in tables

	 division by a one-digit number (simple calculation in which a quotient is a 1- or 2-digit number) Decimal numbers (moved from grade 4) Meaning and representation of decimal numbers, addition and subtraction with decimal numbers (the tenths place) Fractions (moved from grade 4 and 5) Meaning and representation of fractions, simple addition and subtraction with fractions Abacus Representing numbers, addition, subtraction 			Tables and bar graphs	
Grade 4	Representation of whole numbers Rounded numbers • approximate numbers, rounding off, estimating results of the four arithmetic operations (moved from grades 5 and 6) Division with whole numbers • division by a 2-digit number, etc. {Acquisition and utilization of the four arithmetic operations with whole numbers} Calculation with decimal numbers • addition/subtraction with decimal numbers (the tenths and hundredths places, etc.) • multiplication/division with decimal numbers (decimal number, decimal numbers (decimal number × whole number, decimal number ÷ whole number) (moved from grade 5) Calculation with fractions • addition/subtraction of fractions with like denominators (proper fraction, [improper fraction]), etc. (moved from grade 5)	Area • Units of area (<i>cm</i> ² , <i>m</i> ² , <i>km</i> ² , [<i>ac</i> , <i>ha</i>] and measurements • finding areas of squares and rectangles Units of angles (degree)	 relationship of parallelism and perpendicularity of lines (moved from grade 5) parallelogram, rhombus, trapezoid (moved from grade 5) cube, rectangular prism (moved from grade 6) [representing the location of an object] 	Relations between two quantities that co-vary • studying how quantities vary by expressing them in a line graph Representation in mathematical expressions • expressions that have a mixture of four arithmetic operations, expressions with (), formulas • expressions with □, ? Properties of the four arithmetic operations (moved from grade 5) Sorting data • table with two viewpoints, line graphs	a. estimating results of calculations, and making judgments b. thinking about ways to calculate areas and explaining them c. tessellating (tiling) with figures such as parallelograms and studying their properties d. investigating how quantities encountered daily life relate to each other

	Abacus				
	 addition/subtraction 				
Grade 5	 Properties of whole numbers even and odd numbers, <u>divisors</u> and multiples (moved from the six grade), [prime numbers] Notation system for whole numbers and decimal numbers Calculation with decimal numbers multiplication/division with decimal numbers (the tenths and hundredths places, etc.) Calculation with fractions addition/subtraction of fractions with unlike denominators (proper fraction, [improper fraction]), (moved from grade 6) multiplication/division with fractions (fraction × whole number, fraction ÷ whole number) 	 Area finding areas of triangles and parallelograms [finding areas of rhombi and trapezoids] Volume (moved from grade 6) units of volume (cm3, m3) and measurement finding volumes of cubes and rectangular prisms Mean of measurements Per-unit-quantity (example: population density) (moved from grade 6) 	 polygons and regular polygons [congruence of figures] (partially moved from lower secondary schools) properties of geometric figures ratio of circumference of a circle to its diameter (<i>pi</i>) p<u>risms, cylinders (moved from grade 6)</u> 	Simple proportional relations Observation and examination of quantitative relations • studying a relation of two quantities that are represented by a simple mathematical expressions Percentage Pie charts, Bar graphs	a. thinking about ways of calculating and explaining them b. thinking about ways to calculate areas and explaining them c. drawing and constructing congruent figures d. explaining properties of geometric figures inductively and deductively e. selecting a graph or a table depending on each objective and applying them
Grade	Calculations with fractions	Approximate shape	Geometric figures	Ratio	a. thinking about ways of
6	 multiplication / division of fractions ([calculations that involve both fractions and decimal numbers, etc.) {Acquisition and utilization of the four arithmetic operations with decimal numbers and fractions} 	 approximate areas, etc. <u>Area (moved from grade 5)</u> <u>finding area of a circle</u> Volume finding volume of prisms and cylinders (moved from lower secondary school) Speed meaning and representation, finding speed [Metric system, how it works] 	 [scaled drawing] (moved from lower secondary schools) [symmetric figures] (moved from lower secondary schools) 	Proportion and [inverse proportion] (partially moved from lower secondary schools) [Expressions with letters such as <i>a</i> , <i>x</i> (partially moved from lower secondary schools)] Ways to examine data • the mean of data • [frequency distribution] [Possible outcomes (moved from lower secondary schools)]	calculating and explaining them b. investigating relationships among units c. reduced and enlarged drawing; finding symmetric figures d. solving a problem by using the idea of proportion.

Table 2: Structure of the content of mathematics in lower secondary schools

	A. Numbers and Mathematical Expressions	B. Geometric Figures	C. Functions	D. Data Handling	Mathematical activities
Grade 7	 Positive numbers, Negative numbers a. need for and meaning of positive and negative numbers [Set of numbers and four arithmetic operations] b. Meaning of four arithmetic operations with positive and negative numbers c. Four arithmetic operations with positive and negative numbers d. Applications of positive and negative numbers Mathematical Expressions with letters a. need for and meaning of using letters in expressions b. representation of multiplication and division c. addition and subtraction of linear expressions d. representations with inequality] Linear equation with one unknown a. need for and meaning of equations and the meaning of its solution b. Properties of equations and how to solve them c. Solving and using linear equations 	 Planar figures a. basic drawing and its application [b. moving basic figures] Spatial figures a. spatial relationship of lines and planes b. structure of spatial figures and their representation on a plane [{projection drawing}] c. measurements in basic figures [{surface area and volume of a sphere}] 	Proportion, Inverse proportion <u>a. Meaning of functional</u> <u>relationship (moved</u> <u>from the eight grade)</u> b. Meaning of proportion and inverse proportion c. Meaning of coordinates d. tables, mathematical expressions, and graphs of proportion and inverse proportion e. applying proportion and inverse proportion	[Dispersion of data and representative values of data] [a. Need for and meaning of histograms and representative values] [b. applying histograms and representative values]	In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented. a. Based on previous learned mathematics ideas, find the properties of numbers and geometric figures. b. apply mathematics in daily life c. explain and discuss ideas, using mathematical expressions and in one's own words
Grade 8	 Four arithmetic operations of expressions with letters a. addition/subtraction of simple integer expressions, and multiplication/division of monomials b. representing and interpreting expressions with letters c. purposeful transformation of expressions Simultaneous linear equations with two unknowns 	 Basic planar figures and properties of parallel lines a. properties of parallel lines and angles b. properties of angles of polygons Congruence of figures a. congruence of planar figures, congruence conditions of triangles b. the need for and the meaning of proofs, and its method 	Linear function a. phenomena and linear functions b. tables, mathematical expressions and graphs of linear functions c. linear equations and functions with two unknowns d. applying ideas of linear functions	Probabilities a. Need for and meaning of finding probabilities b. applying ideas of probabilities	In learning content within and across domains, opportunities to do mathematical activities such as shown below should be implemented. a. Based on previously learned mathematics ideas, find the properties of numbers and

[] indicates new content. The <u>underline</u> indicates content moved between grades.

	 a. need for and meaning of simultaneous linear equation with two unknowns, and the meaning of the solution b. simultaneous equations and the meaning of its solution c. solving simultaneous equations and applying them 	c. basic properties of triangles and parallelograms			geometric figures, and develop further ideas. b. apply mathematics in daily life c. explain and discuss ideas in logical steps by clarifying reasoning.
Grade 9	Square root a. Need for and meaning of square roots [{rational numbers, irrational numbers}] b. calculations of expressions with square roots c. using square roots Expanding and factoring mathematical expressions a. multiplication/division of polynomials and monomials b. expanding and factoring of simple expressions c. representing and explaining with expressions with letters Quadratic equations a. need for and meaning of quadratic equations, and the meaning of the solutions [b. solving quadratic equations by factorization and completing squares] c. Solving quadratic equations by using the quadratic formula d. applying quadratic equations	 Similarity of geometric figures a. similarity of planar figures, similarity conditions of triangles b. basic properties of geometric figures c. parallel lines and ratios of segments. [d. relations among ratios of similarity, ratios of areas, ratios of volumes] e. applying the properties of similarity of geometric figures Inscribed angle and central angle a. Relationship between the inscribed angle and the central angles, and its proof (moved from eighth grade) [converse of the Central Angle Theorem] b. Application of the relationship between an inscribed angle and a central angle (moved from the seventh grade) Pythagorean Theorem a. Pythagorean Theorem and its proof b. applying the Pythagorean Theorem 	 isosceles triangle, equilateral triangle (moved from grade 4) angle (moved from) circle, sphere (moved from grade 4) 	[Sampling] [a. Need for and meaning of sampling [b. performing sampling.	

Chapter 2. The Objectives and Content of Mathematics

"Section 3: Elementary School Mathematics" in the Course of Study is composed of the following:

- 1. OBJECTIVES OF THE SUBJECT
- 2. OBJECTIVES AND CONTENT IN EACH GRADE
- 3. THE CONSTRUCTION OF TEACHING PLANS AND REMARKS CONCERNING CONTENT IN EACH GRADE

In this chapter, the objectives of mathematics and the objectives for each grade are explained first. Then, the objectives and main content in each of four domains, "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometric Figures," and "D. Quantitative Relations" are explained.

Section 1: The Objectives of Mathematics

1 Objectives of the subject

In the objectives of the subject, skills, capabilities, and attitudes that we need to develop in students through the study of mathematics are outlined. This section clarifies the role mathematics plays in guiding human development in elementary education.

The objectives in teaching mathematics are described below:

Through mathematical activities, students will: (1) acquire fundamental and basic knowledge and skills regarding numbers, quantities and geometric figures; (2) cultivate their ability to consider phenomena from their daily lives with the foresight to generate and organize logical thinking steps to follow through and to represent those phenomena; (3) recognize the joy of mathematical activities and the merit of mathematical manipulation; and (4) foster a willing disposition to make use of mathematics in daily life and studies.

Even though the objectives of each part are closely related to others, each one is discussed in five separate parts to help understand the points of each objective clearly.

(1) Through mathematical activities

The objective begins with the phrase "Through mathematical activities," and it affects the overall teaching objectives of mathematics. It states basic ideas about how to proceed with the course of study to achieve the objectives listed below.

Mathematical activities are various activities related to mathematics where students engage willingly and purposefully.

"Students engage willingly and purposefully" means students try to find new properties or new ways of thinking or try to solve a concrete problem. In order to help students experience the meaning of quantities and geometric figures, raise their ability to think, reason and express ideas, and let them feel the joy and meaning in learning mathematics through mathematical activities, it is necessary to teach them to have a sense of these objectives and to engage them in learning willingly. For this reason, simply listening to what teachers say and doing practice problems are not included in the definition of mathematical activities.

Mathematical activities can include various activities. They are often physical handson, experimental activities, but there are others. Thinking about mathematical problems, developing and applying one's knowledge, and presenting and explaining what they think are all activities that do not deal with concrete objects, but they are included in mathematical activities.

By implementing mathematical activities, the following improvements can be expected in mathematics classes:

- Mathematics classes become more centered on students' activities and more proactive.
- Mathematics classes become more fun to students.
- Mathematics classes become easier to understand to students.
- Mathematics classes become more compelling.
- Mathematics classes foster creativity and a disposition to explore ideas.
- Mathematics becomes more connected to daily life and natural phenomena.
- Mathematics can be more easily related to other subjects and to Integrated Study^{*}.

(2) acquire fundamental and basic knowledge and skills regarding numbers, quantities and geometric figures

The fundamental and basic knowledge and skills students acquire in mathematics, as well as their abilities in language, provide the foundation for daily life and future studies. They form the basis for daily activities and learning in other subjects and Integrated Study. They also form the core of future studies in mathematics. In this regard, they are important.

Knowledge and skills can include: understanding the meaning, concepts, principles, and laws behind quantities and geometric figures; methods for concisely expressing quantities and geometric ideas using simple mathematical expressions and symbols; and methods for using tools to measure quantities or to draw geometric figures. It should also be noted that any knowledge and skills students acquire is often built upon knowledge and skills previously acquired.

What is meant by "acquire" is that students grasp the meaning of quantities and geometric ideas, are conscious of their understanding, and are able to use their knowledge and skills according to their goals in daily life and in their studies.

If students are forced to memorize steps of calculation mechanically, without understanding the meaning, or if teaching is done in a way that only emphasizes the

^{*} A subject in the Course of Study

formal processing of calculations, the values of knowledge and skills are significantly reduced. It is important to instruct students to understand the concept of calculation, and use it appropriately depending upon the objectives.

(3) cultivate their ability to consider phenomena from their daily lives with the foresight to generate and organize logical thinking steps to follow through, and to represent those phenomena

This part of the objective describes the cultivation of the power of thought and expression. In this revision, the phrase, "to represent those phenomena" was added to indicate the ability to express ideas. The ability to think and the ability to express are complementary to one other. In the process of expressing ideas, students may notice good points or find an error, and by expressing their ideas, they often learn to carry out their thinking in logical steps and create better ideas. In class, students present various ideas, and they learn from other students. With this background, the ability to think and the ability to represent are both explicitly stated.

"Phenomena in their daily lives" here means a wide range of phenomena to which mathematics can be applied in their real life situation or learning situation. In these situations, if students encounter some difficulty in the goals they want to achieve, the phenomena become their problems. In order to solve these problems, while finding new ways to get results, students need to have the foresight to devise logical thinking steps.

In attempting to have foresight about the method and results of problem solving, looking at parts or the entire problem and conducting trials and experiments are quite often useful. The inductive method in which various examples are studied and the commonality is sought, and the analogical thinking where results are inferred from the similar cases, can also be used. Having foresight is important to carrying out problem solving steps appropriately and efficiently. To indicate if the steps for solving problems and the results are correct, students are required to organize logical thinking steps. It is based on the idea of processing one step at one time by justifying a fact. The deductive method in which explanation is given based on some assumption is well known, but inductive and analogical reasoning are also considered to be logical thinking methods since students need to indicate facts that they use for reasoning.

The important aims that the subject of mathematics focuses on are to enhance the ability to think in logical steps with foresight in the process of problem solving, reasoning, and making inferences.

These are also considered to be the aims of other subjects. But in mathematics, students frequently encounter more situations where they need to think inductively or deductively. Furthermore, in consideration of the systematic and objective nature of the subject, mathematics can contribute most greatly.

In teaching in each grade, it is important to cultivate the ability to think in organized logical steps with foresight, according to the developmental stages of the students and appropriately within the content of each grade. It is important to implement learning activities where students can express their thoughts and make explanations to their friends by using things such as words, numbers, mathematical expressions, diagrams, tables, and graphs.

(4) recognize the joy of mathematical activities and the merit of mathematical manipulation

This part addresses the affective aspect of the objectives in mathematics. The comparative study of the International Association for the Evaluation of Educational Achievement (IEA) reported that the percent of Japanese students who say they like mathematics is less than in other nations, and this trend has not yet reversed. In teaching mathematics, it is very important to create classes where students can feel the joy, interest, and wonder of mathematics. The statement in the objectives, "recognize the joy of mathematical activities," addresses this point.

There are various mathematical activities such as the following: activities that associate daily life experience with mathematics, hands-on activities that involve making things, experimental activities that can include checking the actual size of numbers and quantities, exploration activities that can include finding hidden rules in the multiplication table, and developmental activities in which new problems are created from the problems solved. The goal is that students will recognize the joy of mathematics though these activities. Students are by nature energetic, and they enjoy activities. It is important to create joyful mathematics classes.

The latter part states "(recognize the) merit of mathematical manipulation." To make students recognize this merit, it is important to create learning activities where students can grasp and process daily phenomena mathematically. To grasp phenomena mathematically is to investigate and to explore, for example, by paying attention to elements such as numbers, quantities, geometric figures in phenomena, focusing on functional ideas such as changes and relationships, and applying the idea of sets in clarifying a target.

To recognize the merit is to recognize the values of mathematics and the meaning of learning mathematics; it can boost a student's motivation to learn and lead to deeper understanding, and it can also foster a more positive attitude towards learning mathematics. In this respect, it is especially important that the teacher's instruction should lead students to be actively concerned about the subject.

In regard to merits, there are merits that are contained in the knowledge and skills of quantities and geometric figures, and there are merits contained in thinking, judgment, and representation. What are these merits? They can be usability, simplicity, generality, accuracy, efficiency, expandability, and beauty.

For example, in mathematics, using numbers, students investigate how many things there are, and compare their size. Learning these types of things can be applied to their daily life situation, and the fact that numbers have a built-in usability is of merit. Whole numbers can be represented by the base-10 notation system. And this notation has the value of expressing a size by the location of each numeral. Through this property, a number can be expressed simply, and two numbers can be compared by size. This merit in the ways of representation is related to usability, simplicity, and generality. As discussed here, it is important to research teaching materials by clarifying the merits in each method, and it is also important to be creative in teaching so that students may notice these merits.

(5) foster a willing disposition to make use of mathematics in daily life and in their studies

This part describes the goal of fostering an attitude in students that encourages them to use mathematics. In this revision, two points are emphasized: *acquiring* fundamental and basic knowledge and skills, and *using* acquired knowledge and skills. Mathematics can be used in various daily life and learning situations. By applying what students have learned in mathematics to various daily life and learning situations, the study becomes meaningful, and students can really experience and appreciate the merits of mathematics. What was meant by "daily life and learning situations" can be interpreted broadly. This could be their family time, school life, and life in their communities, and can also include their future life in society. About learning, of course, what has been learned in mathematics can be used in other subjects, but it is important to apply this knowledge to the future study of mathematics. In mathematics, it is possible to create new knowledge and methods from what was previously learned. Also, for general study, students can engage in exploring various learning activities by applying the knowledge, skills, thinking ability, reasoning ability, and representation ability acquired in mathematics.

2 Objectives of each grade

In mathematics, the content and objectives are indicated in each grade. In the objectives of each grade, it primarily describes the main points that teaching should focus on. Since each grade has the four domains, "A. Numbers and Calculations," "B. Quantities and Measurements, " "C. Geometric Figures," and "D. Quantitative Relations," objectives in each grade are categorized by the domains.

The Objectives of Grade 1

- (1) Through activities such as those involving concrete objects, students will enrich their sense for numbers. They will understand the meaning and representations of numbers. They will also understand the meaning of addition and subtraction, investigate ways to carry out calculations and develop the ability to use them appropriately.
- (2) Through activities such as those involving concrete objects, students will enhance experiences that will form the foundation for understanding the ideas of quantities and their measurements, as well as enriching their sense for measurements.
- (3) Through activities using concrete objects, students will accumulate experiences that will form the foundation for understanding geometry, as well as enrich their sense of geometry.
- (4) Through activities using concrete objects, students will be able to represent numbers, quantities, and their relationships by using words, mathematical expressions, and diagrams, and will be able to interpret such representations.

In "A. Numbers and Calculations" in grade 1, the meaning and representation of numbers up to 2-digit numbers are taught. As a bridge between grades, 3-digit numbers are taught in simple cases. In calculation, addition between two 1-digit numbers and the inverse of this operation^{*} are taught so that the students acquire the skills of adding and subtracting accurately and reliably. As a bridge between grades, simple cases of sums and differences involving 2-digit numbers are taught.

In "B. Quantities and Measurements," students are taught to directly compare length, area, and volume. The aim is for this to become a fundamental learning activity that will form the basis for learning units and measurements. Also, students will be taught to how to tell time in their daily lives.

In "C. Geometric Figures," students are taught about the shapes of objects in daily life, and both planar and solid figures are dealt with. The aim is that understanding figures can become a learning activity that will provide the basis for learning geometric figures.

In "D. Quantitative Relations," students are taught to represent cases where addition and subtraction can be applied by using mathematical expressions, and to represent the number of objects in diagrams or in pictures.

The Objectives of Grade 2

- (1) Through activities such as those involving concrete objects, students will enrich their sense for numbers. They will deepen their understanding of the meaning and representations of numbers, addition and subtraction operations, and applications. Students will understand the meaning of multiplication, investigate ways to carry out calculations, and develop the ability to use them appropriately.
- (2) Through activities such as those involving concrete objects, students will understand the process of measuring and the units for length and volume. They will further enrich their sense for measurements.
- (3) Through activities using concrete objects, students will be able to understand geometric figures such as triangles and quadrilaterals, and enrich their sense of geometry.
- (4) Through activities using concrete objects, students will be able to represent and interpret numbers and their relationship to quantities by using words, numbers and mathematical expressions, figures, tables and graphs.

In "A. Numbers and Calculations," the meaning and representation of numbers up to 4 digits is taught, as well as 1 *man* [10000]. Simple fractions such as 1/2 and 1/4 are taught, and the aim is for learning activities here to become the basis for learning fractions. In calculation, the main focus is on addition of two 2-digit numbers and on the

^{*} [Translators' note: i.e. subtraction, including cases of (2-digit) – (1-digit) = (1-digit).]

inverse of this operation^{*}, and on basic multiplication (ku-ku) [up to 9×9]; therefore these skills can be solidly acquired. As a bridge between grades, simple cases of addition and subtraction of 3-digit numbers are taught, and multiplication between 2-digit and 1-digit numbers is also taught.

In "B. Quantities and Measurements," units of length (e.g. meters) and their measurement, and units of volume (e.g. liters) and their measurement are taught. Telling time (on a clock) is also taught.

In "C. Geometric Figures," students are taught about triangles, quadrilaterals, squares, rectangles, and right triangles as plane figures, and the box as a solid figure.

In "D. Quantitative Relations," students are taught to represent the relationship between addition and subtraction, and cases in which to apply multiplication by using mathematical expressions. Also, students are taught to represent data by using simple tables and graphs.

The Objectives of Grade 3

(1) Students will be able to use addition and subtraction appropriately. They will further deepen their understanding of multiplication and use it appropriately. They will also understand the meaning of division, investigating ways to carry out calculations as well as developing the ability to use them appropriately.

Furthermore, they will understand the meaning and the representations of decimal numbers and fractions.

(2) Students will understand the process of measuring and the units for length, weight, and elapsed time.

(3) Students will be able to understand geometric figures such as isosceles triangles and equilateral triangles by paying attention to the elements that compose geometric figures.

(4) Students will be able to represent and interpret numbers, quantities, and their relationships by using words, numbers, and mathematical expressions as well as diagrams, tables, and graphs.

In "A. Numbers and Calculations", students are taught about the unit of *man* [tenthousand] and the relative size of numbers as a representation of whole numbers. In calculation of whole numbers, the following processes are taught: addition and subtraction of 3- and 4-digit numbers, multiplication of 2- and 3-digit numbers by 1- or 2-digit numbers and division in which both the divisors and the quotients are 1-digit. As a bridge between grades, simple cases of addition and subtraction of 3-digit numbers are taught. Multiplication between 2-digit numbers and 1-digit number is also taught. Teaching about the meaning and representation of decimal numbers and fractions begins formally from grade 3. In grade 2, they are exposed to some basic materials, so teaching should be done using these experiences. Furthermore, addition and subtraction of decimal numbers using tenths and simple cases of addition and subtraction of

^{*} [Translators' note: i.e. subtraction, including cases of (3-digit) – (2-digit) = (2-digit).]

fractions are taught. Also, students are taught to represent numbers on the abacus and to add and subtract using the abacus.

In "B. Quantities and Measurements," units of length (kilometers) and their measurement, and units of weight (e.g. grams) and their measurement are taught. Also, students are taught about units of time (seconds) and how to determine clock time and elapsed time.

In "C. Geometric Figures," students are taught about isosceles triangles, equilateral triangles, and circles as plane figures, and spheres as solid figures.

In "D. Quantitative Relations," students are taught how to represent cases where division may be applied by using mathematical expressions; to represent quantitative relationships in mathematical expressions by using a symbol such as \Box ; and to interpret and draw bar graphs.

The Objectives of Grade 4

- (1) Students will deepen their understanding of division and use it appropriately. Students will further deepen their understanding of the meaning and the representations of decimal numbers and fractions, as well as addition and subtraction of decimal numbers and fractions, investigating ways to carry out calculations. Moreover, students will understand approximate numbers and use them appropriately.
- (2) Students will understand the process of measuring and the units of measurement for area and will be able to determine the area of geometric figures. They will also understand the process of measuring angles and the unit of measurement for angles.
- (3) Students will be able to understand plane figures such as parallelograms and rhombuses and solid figures such as rectangular prisms by paying attention to the elements that compose the geometric figures and the relationships among those elements.
- (4) Students will be able to represent and investigate numbers, quantities, and their relationships by using words, numbers, and mathematical expressions as well as diagrams, tables, and graphs.

In "A. Numbers and Calculations," students are taught the units of *oku* [hundred million] and *cho* [trillion] as representations of whole numbers.

In the calculation of whole numbers, division of 2- and 3-digit numbers by 1- and 2digit numbers is taught. In grade 4, the four arithmetic operations of whole numbers are completed and summarized, and teaching should be done in a way that solidifies students' calculation skills of whole numbers and enhances their ability to apply their skills.

In decimal numbers, calculation of the sum and the difference of decimal numbers is taught. Also, multiplying and dividing decimal numbers by whole numbers is taught.

Addition and subtraction of fractions with like denominators is taught.

Furthermore, students are taught to understand approximate numbers and to estimate the results of the four arithmetic operations. Also, students are taught about addition and subtraction using the abacus.

In "B. Quantities and Measurements," units of area (e.g. square meters) and their measurements, as well as how to determine the area of squares and rectangles are taught. Also, students are taught about the unit of angle measure (degree) and how to measure angles.

In "C. Geometric Figures," students are taught about parallelograms, rhombuses, and trapezoids as plane figures, and cubes and rectangular prisms as solid figures. Also, students are taught about how to represent the position of an object in space.

In "D. Quantitative Relations," students are taught about how quantities vary, broken-line graphs, mathematical expressions that represent the relationship between two quantities, properties of the four arithmetic operations, and how to sort, organize, and represent data.

The Objectives of Grade 5

- (1) Students will deepen their understanding of the properties of whole numbers. They will also deepen their understanding of the meaning of multiplication and division of decimal numbers as well as addition and subtraction of fractions, investigating ways to carry out calculations, and developing the ability to use operations and properties appropriately.
- (2) Students will be able to determine the area of polygons, such as triangles and parallelograms, and the volume of rectangular prisms. Students will understand the mean of measured quantities and the ratio of two unlike quantities.
- (3) Students will deepen their understanding of plane figures and be able to understand solid figures such as prisms.
- (4) Students will consider quantitative relationships.

In "A. Numbers and Calculations," even and odd numbers, and factors and multiples are taught as the properties of whole numbers. Also, the notation system for whole numbers and decimal numbers is taught. With decimal numbers, multiplication and division are taught. With fractions, addition and subtraction with unlike denominators are taught.

In "B. Quantities and Measurements." ways to determine the area of triangles, parallelograms, rhombuses, and trapezoids are taught. Also, in this grade, students are taught about units of volume (e.g cubic meters) and their measurements, and ways to determine the volume of cubes and rectangular prisms. Furthermore, the mean of measurements and per-unit quantities are dealt with.

In "C. Geometric Figures," polygons and regular polygons are taught as plane figures, and cubes and rectangular prisms as solid figures. Also, the congruence of figures and recognition of the properties of geometric figures are taught in this grade.

In "D. Quantitative Relations," simple cases of proportional relationships are taught. This will provide the basis for understanding proportion in grade 6. Also, mathematical expressions that express quantitative relationships, and percentage and pie charts, are taught.

The Objectives in Grade 6

- (1) Students will deepen their understanding of the meaning of multiplication and division of fractions, investigate ways to carry out the calculations, and use them appropriately.
- (2) Students will be able to determine the area of circles as well as the volume of prisms. They will also understand and be able to determine speed.
- (3) Students will understand scale drawings, symmetric figures, and deepen their understanding of geometric figures.
- (4) Students will understand ratio and direct proportion, using their understanding to consider quantitative relationships, and represent the relationships in mathematical expressions by using letters. Furthermore, students will investigate the distribution of data and be able to consider it statistically.

In grade 6 "A. Numbers and Calculations," multiplication and division of fractions is taught.

Also, the learning of the four arithmetic operations of decimal numbers and fractions is completed, and teaching should done in the way that solidifies students' calculation skills of these types of numbers and that enhances their ability to apply these skills.

In "B. Quantities and Measurements," by measuring geometric figures, students are taught how to approximate shapes and areas, how to determine the area of a circle, and how to determine the volume of prisms and cylinders. Ways of determining speed and the system of the metric units are also taught in this grade.

In "C. Geometric Figures," students are taught about scaled drawing and symmetric figures.

In "D. Quantitative Relations," students are taught about ratios, proportion and inverse proportion, and mathematical expressions with letters. In data handling, mean, frequency distribution, and analysis of all possible outcomes for actual events are taught in this grade.

Section 2: The Content of Mathematics

1 Structure of the Content

The contents consist of four domains: "A. Numbers and Calculations," "B. Quantities and Measurements," "C. Geometric Figures," and "D. Quantitative Relations." They are configured in this way to make it easier to see the overall structure and to clarify systematic and developmental paths. In this revision, "mathematical activities" is added following the four domains.

The three domains, A, B, and C, represent respectively, numbers, quantities, and geometric figures, which are targets of learning in mathematics.

The domain "A. Numbers and Calculations" consists of the contents that describe the meaning and representation of numbers such as whole numbers, decimal numbers, and fractions, and methods of calculation.

The domain "B. Quantities and Measurements" is mainly about units and measurements of quantities surrounding us in daily life.

The domain "C. Geometric Figures" is mainly about the meaning and properties of plane figures and solid figures, and the structures of geometric figures. In each of these domains, it is important to conduct teaching by closely connecting understanding of the meaning of numbers, quantities, and geometric figures with activities such as calculation, measurement, and composition.

The domain "D. Quantitative Relations" involves ways of thinking and methods that can be commonly used in handling quantities and geometric figures. In this domain, the main contents include some ideas of functions such as change and correspondences, representation by mathematical expressions, and tables and graphs.

2 Overview of the contents of each domain

A. Numbers and Calculation

(1) Objectives of "A. Numbers and Calculations"

In this domain, the objectives are to understand the meaning and representation of whole numbers, decimal numbers, and fractions, and to have a rich sense of numbers. Also, students will understand the meaning of calculations with whole numbers, decimal numbers, and fractions and learn to think about ways to calculate with them, acquire calculation skills and the ability to apply them. Additionally, it is also an important object in this domain for students to recognize the joys of mathematical activity and the merits of mathematical manipulation by acquiring the ability to think mathematically.

(2) Overview of the contents of "A. Numbers and Calculations"

The table below shows the main contents in each grade categorized by numbers and calculation separately.

Grade	Numbers	Calculations
Grade 1	 2-digit numbers Simple 3-digit numbers	 Addition of 1-digit numbers and its inverse, subtraction Addition/subtraction of simple 2-digit numbers.
Grade 2	 4-digit numbers (numbers up to 1 <i>man</i> [10000]) Base-ten notation system Simple fractions 	 Addition and its inverse, subtraction of 2-digit numbers Addition/subtraction of simple 3-digit numbers. Multiplication table (<i>ku-ku</i>) Multiplying simple 2-digit number by 1-digit number
Grade 3	 Units of <i>man</i> (numbers up to 1 <i>oku</i> [100,000,000]) Decimal numbers (tenths) Fractions 	 Addition/Subtraction of whole numbers (3-digit, 4-digit numbers) Multiplication of whole numbers (2-digit numbers, 3-digit numbers) Division of whole numbers (divisors and remainders are 1-digit numbers) Simple division (remainders are 1-digit numbers and divisors are 2-digit numbers) (mental calculation) Calculation by using an abacus Addition/subtraction of simple decimal numbers and fractions.
Grade 4	 Units of <i>oku</i> [100 million] and <i>chou</i> [1 trillion] Approximate numbers Decimal numbers Fractions (proper fraction, improper fraction, mixed fraction) 	 Division of whole numbers (Divisors are 1-digit numbers or 2-digit numbers, and dividends are 2-digit or 3-digit numbers) Estimating results of calculations (simple mental calculation) Acquiring calculation skills of whole numbers Calculation by using an abacus Addition/subtraction of decimal numbers in which multipliers and divisors are whole numbers Addition/subtraction of fractions with like denominators.
Grade 5	 Even numbers, odd numbers Divisors, multiples (greatest common divisor, least common multiple) Prime numbers 	 Multiplication and division of decimal numbers in which multipliers and divisors are decimal numbers Addition/subtraction of fractions with unlike denominators Multiplication and division of fractions in which multipliers and divisors are whole numbers
Grade 6	• Reciprocal	 Multiplication and division of fractions in which multipliers and divisors are fractions Acquiring calculation skills of decimal numbers and fractions

In grade 1, the meaning and representation of 1-digit and 2-digit numbers are taught. As a bridge between grades, simple cases of 3-digit numbers are also taught. In calculation, additions of 1-digit numbers and its inverse, subtractions of 1-digit numbers are taught, and it is necessary that students acquire these calculation skills. As a bridge between grades, addition and subtraction of 2-digit numbers are also taught.

In grade 2, the meaning and representation of numbers up to 4-digits is taught. The unit *man* [10,000] is also taught. Simple fractions such as 1/2 and 1/4 are taught in this

grade, and should become the foundation for learning fractions in the future. In calculations, the main contents are in the addition and subtraction of 2-digit numbers and the multiplication table (*ku-ku*). It is necessary that students acquire these calculation skills. As a bridge between grades, simple cases of addition and subtraction of 3-digit numbers and multiplication between 2-digit and 1-digit numbers are taught in this grade.

In grade 3, as representations of whole numbers, the units of *man*, and the relative sizes of numbers are taught. In calculation of whole numbers, students are taught addition and subtraction of 3-digit and 4-digit numbers, multiplying 2-digit and 3-digit numbers by 1-digit or 2-digit numbers, and division where both divisors and reminders are 1-digit numbers. Learning about the meaning and representation of decimal numbers and fractions begins in earnest in grade 3. In grade 2, students have done learning activities on the basics of fractions, and teaching should make use of those experiences. Furthermore, addition and subtraction of decimal numbers in the tenths is taught in this grade. Also, the use of the abacus to represent numbers and to do addition and subtraction is taught.

In grade 4, as representations of whole numbers, the units of *oku* [hundred thousand] and *chou* [trillion] are taught. In the calculation of whole numbers, students are taught about division in which divisors are 1-digit or 2-digit numbers and dividends are 2-digit or 3-digit numbers.

Grade 4 is where the study of the four arithmetic operations of whole numbers is complete and to be reviewed, so students should securely acquire "calculation skills of whole numbers and enhance their abilities to apply these skills." Addition and subtraction, and multiplication and division with multipliers and divisors that are whole numbers, are taught for decimal numbers. Regarding fractions, addition and subtraction of fractions with like denominators are taught. Furthermore, students are taught to understand approximate numbers and to become able to estimate the results of their calculations.

Also, addition and subtraction using an abacus is taught.

In grade 5, even numbers, odd numbers, multiples and divisors are taught as properties of whole numbers. Also, students are taught about the notation system for whole and decimal numbers.

Multiplication and division are taught for decimal numbers. In the case of fractions, addition and subtraction of fractions with unlike denominators is taught.

In grade 6, multiplication and division of fractions are taught. Grade 6 is where the study of the four arithmetic operations of decimal numbers and fractions is complete and to be reviewed, so students should securely acquire "calculation skills with decimal numbers and fractions."

(3) Main contents

(1) Numbers

a. Whole numbers

Whole numbers are used to represent a number of things or to represent an order in a series. Whole numbers are represented by the base-ten notation system. In grade 1 they are simply called "numbers" in teaching. Beginning in grade 3 they are referred to as "whole numbers" to distinguish them from decimal numbers and fractions.

In grade 1, students are taught to compare numbers of objects in groups by making one-to-one correspondence. They are taught to correctly count or represent the number and order of objects, to put numbers in sequence, and to consider a number as a sum or difference of other numbers. Students are expected to understand the meaning of whole numbers.

In grade 2, students are taught to count objects by rearranging them into groups of the same size or by classifying them, to represent numbers in the base-ten notation system, to treat 10 and 100 as units, and to consider a number as a product of other numbers. Through these activities, students are expected to enhance their abilities to use whole numbers.

In grade 3, students are taught about the unit of *man* [10,000] and the workings of the base-ten notation system. Students are expected to deepen their understanding in the representation of whole numbers, and to enhance their ability to use numbers. Also, they are introduced to the concept of rounding and learn about approximate numbers, and students are expected to use these skills appropriately.

In grade 4, students are taught about the units of *oku* [hundred million] and *chou* [trillion] to deepen their understanding of the base-ten notation system.

In grade 5, even and odd numbers and multiples and divisors are taught to deepen their understanding of the properties of whole numbers.

b. Decimal numbers and fractions

In grade 3, decimal numbers are used only to represent the size of fractional parts. Ways to represent decimal numbers with one decimal place are taught. In grades 4 and 5, additional place values such as the hundredths and the thousandths are taught and students will deepen their understanding about how decimal numbers are represented in the base-10 notation system.

Teaching about fractions begins in grade 2. In this grade, simple fractions such as 1/2 and 1/4 are taught and these learning activities become the basis for the future understanding of fractions.

In grade 3, students are taught to use fractions to express an amount obtained as a result of equal partitioning and a fractional part of a number. They are taught to become aware that a fraction can be considered as a collection of units; for example, 4/3 is four units of 1/3.

In grade 4, students are taught to be aware that some fractions are equal in size in simple cases. Also, they are taught that a fraction that is less than 1 is called a *proper*

fraction, a fraction that is greater than or equal to 1 is called an *improper fraction*, and an improper fraction represented by the sum of a whole number and a proper fraction is called a *mixed number*.

In grade 5, students are taught to express the results of dividing a whole number by another whole number as a fraction. For example, $2\div 3 = 2/3$. Also, expressing decimal numbers in fraction form and comparing the size of fractions with unlike denominators is taught with the expectation that students' understanding of fractions will be deepened.

(2) Calculations

Understanding the meaning of calculations, having the ability to think about methods of calculation, acquiring calculation skills, and the ability to apply them are considered to be important objectives in teaching calculations

a. Addition and subtraction

Addition and subtraction of whole numbers. In grade 1, students are taught to become aware of situations where addition and subtraction may be applied. Students are taught to think about ways of adding and subtracting 1-digit numbers, and they are expected to become able to do the calculations with confidence. In grade 2, in addition to deepening their understanding about situations where addition and subtraction are applied, students are taught to investigate the properties of these operations, and by using those properties students are expected to think about the method of calculation and to verify the results. Also, students are expected to become able to do addition and its inverse, subtraction, of numbers up to 2-digits. In grade 3, students should be able to do addition and subtraction of whole numbers with confidence, and they are expected to have the ability to use addition and subtraction appropriately.

Addition and subtraction of decimal numbers. In grade 3, students are taught to understand the meaning of addition and subtraction of decimal numbers up to the tenths digits; they are expected to think about methods of calculation and to be able to do the operations. In grade 4, students deepen their understanding of addition and subtraction of decimal numbers and they are expected to do the operations with confidence.

Addition and subtraction of fractions. In grade 3, students are taught to understand the meaning of addition and subtraction of fractions in simple cases and to think about the method of calculation. In grade 4, students are taught to understand the meaning of addition and subtraction of fractions with like denominators, and to be able to apply the operations. In grade 5, students are taught to understand the meaning of addition and subtraction of fractions. For students to understand these operations, it is necessary to teach how to find a common denominator.

b. Multiplication and division

Multiplication and division of whole numbers. In grade 2, students are taught to become aware of situations where multiplication and subtraction may be applied, and to understand the meaning and the method of calculation. For example, students can
understand multiplication as the operation to determine the total number when given the number in a group and the number of groups, or the amount that is so many times as many as a base amount. They can determine the product by repeated addition. The multiplication table (ku-ku) is taught, and it is important that students are able to multiply 1-digit numbers with confidence. Furthermore, to deepen their understanding of multiplication, students are taught simple cases of multiplication between a 2-digit number and a 1-digit number.

In grade 3, students are taught to become able to securely multiply 2-digit numbers. Students are also expected to understand the meaning of division and the situations in which division applies. Division includes the case of *partitive* division and the case of *quotative* division. *Partitive division* finds one part of an equally divided number or quantity. *Quotative division* finds how many times one quantity is of another quantity. The method of calculation for these two are identical and, therefore, are treated as one operation. Students are taught to become aware that division is the inverse of multiplication, and they are taught to check results of division by multiplication. By doing, so, students are expected to deepen their understanding of multiplication and division, and to become able to do these calculations securely.

In grade 4, it is important that students become able to do divisions whose divisors are 2 digit numbers.

Multiplication and division of decimal numbers. In grade 4, students are expected to become able to think about the ways of multiplying and dividing decimal numbers when the multipliers and divisors are whole numbers, and to become able to do these calculations.

In grade 5, students are expected to become able to multiply and divide when the multipliers and divisors are decimal numbers. When the multipliers are decimal numbers, students can no longer treat multiplication as repeated additions, so it is necessary to extend the meaning of multiplication.

Multiplication and division of fractions. In grade 5, students are expected become able to think about multiplication and division of fractions when the multipliers and divisors are whole numbers. In grade 6, students are expected to do multiplication and division when the multipliers and divisors are fractions. In dealing with the cases in which multipliers and divisors are fractions, the extended meanings of multiplication and division of decimal numbers can be directly applied, and it is necessary to consider how they are related. Also, students are taught to consider division as multiplication by the reciprocal, and to become aware that multiplications of decimal numbers and fractions.

c. Approximate Numbers and Estimates

In grade 4, students are taught about the meaning of approximate numbers and rounding off. Also, they are taught to estimate results of the four arithmetic operations appropriate to particular objectives.

Approximate numbers and estimates relate to various contents taught after grade 4. It is important that students become able to estimate the results of calculations, and use it to judge the accuracy and the appropriateness of calculations.

B. Quantities and Measurements

(1) Objectives of "B. Quantities and Measurements"

The main objectives of the domain are to gain understanding about units and measurements of various quantities which have close ties with students' daily life, to develop the skills to measure, and to foster a rich sense of quantities.

Quantities taught in mathematics include length, area, volume, time, weight, angle, and speed.

In the study of mathematics, sometimes students are invited to grasp the quantity relations. Units and measurements appropriate for each quantity are taught. Students are taught to become aware of usefulness of expressing size by using units, and they should become able to choose a unit appropriate to each objective.

(2) Overview of the contents of "B. Quantities and Measurements"

The table below shows the main contents in each grade categorized in two by units of quantities, and comparisons/measurements/etc. of quantities

	Units of quantities	Comparisons/Measurements/etc.
Grade 1		Direct comparisons in length, area, and volumeClock reading
Grade 2	 Units of length (<i>mm, cm, m</i>) Units of volume (<i>ml, dl, l</i>) Units of time (day, hour, minute) 	Measurements of length and volume
Grade 3	 Units of length (<i>km</i>) Units of weight (<i>g</i>, <i>kg</i>) [<i>t</i>] Units of time (second) 	 Measurements of length and weight Measuring by choosing appropriate units Calculation of clock time and elapsed time
Grade 4	 Units of area (cm², m², km²) [ac, ha] Units of angle (degree) 	Determining area (square, rectangle)Measurements of angle
Grade 5	• Units of volume (<i>cm</i> ³ , <i>m</i> ³)	 Determining area (parallelograms, trapezoids, rhombuses) Determining volume (cubes, rectangular prisms) Mean of measurements Determining per-unit quantities
Grade 6		 Approximating shapes and approximate area Determining area (circles) Determining volume (prisms, (cylinders)

Many contents in "B. Quantities and Measurements" are closely related to contents in other domains.

For example, to express quantities, knowledge of whole numbers, decimal numbers, and fractions is needed. Area and volume are measurements of solid figures. Furthermore, mathematical expressions needed to determine area and volume are related to the concepts of proportion and functions. It is important to give special consideration to relations with contents in other domains in teaching these contents.

In grade 1, the main objective is to provide students with rich experiences that will become the basis for learning quantities and measurements. Students are taught to compare length, area, and volume, and to use objects around them as units of measure

and to compare sizes by considering how many such units there are. Also, students are taught to read clocks in daily life.

In grade 2, regarding length and volume, students are taught about the meaning of standard units that are shared by everyone, and measurement using the standard units. As units of length, *millimeter (mm), centimeter (cm),* and *meter (m)* are taught; as units of volume, *milliliter (ml), deciliter (dl),* and *liter (l)* are taught. As units of time, *day, hour, minute,* and the relationship among them are taught.

In grade 3, as a unit of length, *kilometer* (*km*) is taught. As units of weight, *gram* (*g*) and *kilogram* (*kg*) are taught. A unit of time, *second*, and calculation of time and elapsed time are taught.

In grade 4, regarding area, students are taught to understand the meaning of measuring area and the units used to express area. As units of area, *square centimeter* (cm^2) , *square meter* (m^2) , and *square kilometer* (km^2) are taught. Students are taught about ways to determine the area of planar figures (squares, rectangles).

Students are taught to consider the size of an angle as a rotation. A unit of angle measure (degree) is taught.

In grade 5, students are taught to think about the ways to determine area of triangles, parallelograms, rhombuses, and trapezoids. Units of area (cm^2 , m^2 , km^2) are taught.

Students are taught to understand the meaning and units of volume. Units of volume (cm^3 , m^3) are taught and students are taught to think about the ways to determine volume of solid figures (cubes, rectangular prisms). Also, the mean of measurements and per-unit quantities (ratio of two different quantities)—for example, population density—are taught in this grade. Also, students are taught to think about the ways to determine the area of a circle.

In grade 6, students are taught to estimate the area of shapes in their surroundings by approximating them with familiar figures. Regarding volume, students are taught to think about the ways to determine the volume of prisms and cylinders. Students already learned about per-unit quantities in grade 5; in grade 6 students begin studying speed. Also in grade 6, students review units of quantities they have studied, and are expected to understand the how the metric system works.

(3) Remarks on the main contents

a. Quantities taught in mathematics

Length is an easy quantity to grasp visually. In grade 1, students are taught ideas such as direct comparison of lengths. In grade 2, students are expected to understand the meaning of standard units that are used by many people. The units of length *millimeter* (*mm*), *centimeter* (*cm*), and *meter* (*m*) are taught and measurement by these units is taught. In grade 3, the unit of length *kilometer* (*km*) is taught. Also, students are taught to become able to choose appropriate units and tools to measure quantities.

Area is the size of a surface that extends. In grade 1, students are taught to directly compare area by activities such as overlapping actual objects. In grade 4, units of area *square centimeter* (cm^2), *square meter* (m^2), and *square kilometer* (km^2) and how to measure area are taught. Also, students are taught to think about the ways to determine area of

planar figures (squares, rectangles). In grade 5, based on what students have learned in grade 4, they are expected to think about the ways to determine area of triangles, parallelogram, rhombus, and trapezoid. In grade 6, students are taught to estimate the area of shapes in their surroundings by approximating them with familiar figures. Also, students are taught to think about the ways to determine area of a circle.

Volume is considered as a quantity of containers such as boxes that students can find in their daily lives. In grade 1, students are taught to directly compare volume, for example, by overlapping objects. In grade 2, the units of volume *milliliter* (*ml*), *deciliter* (*dl*), *liter* (*l*) and how to measure volume are taught.

In grade 5, the units of volume *cubic centimeter* (cm^3), *cubic meter* (m^3) and how to determine volume are taught. Also, students are taught to think about the ways to determine the volume of solid figures (cubes, rectangular prisms).

In grade 6, by using what students learned to grade 5, students are taught to think about the ways to determine volume of prisms and cylinders.

Time is taught by relating to events and experience in daily life. In grade 1, students are taught to become able to read the clock. In grade 2, the units of time *days*, *hours*, and *minutes* are taught and students are expected to understand relations among them. In grade 3, the unit of time *seconds* is taught.

Weight is a quantity that is impossible to grasp visually, and it is important to have activities in which students can actually experience it—for example, by having an object in hand or wearing it. In grade 3, units of weight *gram* (*g*) and *kilogram* (*kg*) and how to measure weight are taught.

Angle is considered as an amount of rotation. In grade 4, the unit of angle measure *degree* and how to measure angles are taught. Angles that relate to shapes are in the contents of "C. Geometric Figures" in grade 3.

Quantities that can be considered as ratio between two quantities, such as speed and population density, are taught in grades 5 and 6.

Population density is a quantity that is expressed as a ratio between two quantities, a number of people and area, and it can be considered as a per-unit quantity.

Speed, taught in grade 6, is a quantity that is a ratio between distance and time, and it can be considered as a distance traveled per a unit time.

b. Meaning and properties of quantities

A quantity expresses size. A number of objects can be expressed in whole numbers, for example by counting them. On the other hand, in measuring length of strings or weight of water, the quantity can be divided infinitely and cannot always be expressed in whole numbers.

There are various types of quantities, such as length, area, volume, weight, angle, and speed. In introducing each quantity for first time, it is important to have activities in which the quantities of actual objects are compared. By actually comparing them, it becomes easier for students to grasp what is being compared, and the meaning of the quantity will become gradually clear to students.

When comparing size, it will help students understand the meaning of quantities if terms describing each quantity are used. For length, for example, "long/short"; for area,

"wide/narrow"; for volume, "large/small" (more/less); for weight, "heavy/light"; for angle, "large/small"; for speed, "fast/slow."

Quantities such as length, area, volume, weight and angle have the following properties:

Imagine that there is a container with 2 liters of juice in it. Even if we move this juice to some other container, or divide it into several containers, the overall amount of the juice does not change and remains 2 liters. Reshaping something or moving it to other places, or dividing it into several groups does not change the overall amount of it. We call this property "preservation of quantity."

Based on the preservation of quantity, additivity is verified. If we add 300 g of clay to 500 g of clay, it becomes 800 g of clay. This illustrates additivity in weight.

Also, if we cut a parallelogram into two and make a rectangle by combining the two parts, we can explain that the area of the original parallelogram is equal to the area of the rectangle. In the explanation, "preservation of quantity" and "additivity" are applied.

c. Comparing size of quantities

With regard to quantities that have fundamental properties, there are stages in teaching: direct comparison, indirect comparison, comparison using arbitrary units, and comparisons using standard units.

In direct comparisons, two sizes are directly compared. For example, when we compare the length of two pencils A and B, we align them at one end and judge relative length from the position of the other ends.

In indirect comparisons, the sizes of A and B are indirectly compared, using some other thing which is the same size as A (or B). For example, if we want to compare the adjoining two sides of a rectangular table, we put a tape along one of the sides, A, that we want to compare, cut the tape at the exact length of A, then place the tape along the other side, B, for comparison.

By doing direct comparisons or indirect comparisons it should become clear to students which quantity among many attributes of a concrete object they are trying to compare.

In comparisons using arbitrary units, the size of A and B are measured using an object possessing the appropriate attribute to find out how many times of that object A and B are. For example, to represent the length of a side of a table, take a pencil, and, if the side of a table is four times as long as a pencil, and the other side is six times as long as a pencil, one side is longer than the other by twice a pencil's length.

So, as illustrated in this example, by quantifying the sizes of objects, comparisons can be made more clearly.

If a quantity just needs to be quantified, arbitrary units are sufficient. But in society, the size of units should be clear to anyone, so students should be taught to become aware that common units are necessary. So, in comparison using standard units, A and B are measured by using standard units that are commonly used.

d. Units of quantities

A unit is a basic size that is used to express size of a quantity.

Size of an object can be expressed in terms of how many multiples of the unit the object is. For example, with length, 3 *m* is three of 1 *m*; with volume, 2/3 l is 2/3 of 1 liter.

A unit of volume, a cubic meter (m^3) is based on a unit of length, meter (m). One cubic meter is the volume of a cube whose edge is 1 meter. Units that are created by multiplying or dividing basic units are sometime called *derived units*.

Speed and population density are expressed as ratios between two different quantities and are examples of derived units. Units can be determined freely, but it is more convenient if we use units that many people commonly use. Such units are called standard units or fundamental units. The meter for length and the liter for volume are examples of standard units. In Japan, they are set by the law called "measurement law"; globally they are determined by the International System of Units.

e. Measurements of quantities

Measurement means to investigate and to determine the size of a quantity. Sometimes measurement tools are used for measuring. To measure length, a ruler or a measuring tape can be used, and to measure the volume of a liquid, a measuring cup can be used. It is important that students be able to choose an appropriate unit and tool according to the size of the quantity they are measuring.

The area of a plane figure is expressed as the number of a basic unit (such as 1 square centimeter) or expressed as how many times as many (times as much) as the basic unit. To determine area, a tool cannot be used; it requires calculations using quantities such as the length of a side. Determining volume of solid figures works the same way.

f. Sense of the sizes of quantities

One of the objectives of teaching quantities and measurements is to enrich a sense of the sizes of quantities. It is important for students to have good sense of the sizes of various quantities and apply this rich sense appropriately. For example, if we take the example of length, we can illustrate this idea in the following:

- When looking at a pencil, estimate its length by saying something like, "It is about 20 cm."
- Choose an appropriate unit and measurement tool for the object to be measured, such as: "To measure this, I can use a 30 cm ruler."
- Approximate the size of a basic unit: "1 meter is about this much."
- Understand the approximate size of concrete objects based on things students find in daily life: "The diameter of 1 yen coin is about 2 cm."

In teaching, it is important to implement hands-on and experiential activities where students can investigate and verify the size of concrete objects, and develop a rich sense of quantities. Also, it is effective to conduct activities involving comparisons and measurements in a variety of situations.

C. Geometric Figures

(1) Objectives of "C. Geometric Figures"

In this domain, the main objectives are to understand the meaning and properties of plane figures and solid figures, to enrich one's sense of geometric figures, and to acquire the ability to think and express ideas mathematically through the process of identifying and explaining properties of geometric figures.

The sense of geometric figures means an ability to recognize the shapes of objects and a sense of the characteristics of a shape and of its properties. It is important to enrich the sense of geometric figures through activities that include observation and construction.

(2) Overview of the contents of "C. Geometric Figures"

The table below shows the main contents in each grade.

Grade	Understanding geometric figures	Components of geometric figures	Viewpoint and methods for investigation of geometric figures
Grade 1	• Shapes of objects from students' everyday lives		 Activities involving observation and composition Words such as "front and back," "right and left," and "above and below"
Grade 2	 Triangles, Quadrilaterals Squares, Rectangles, Right triangles Box-shaped objects 	• Straight line, right angle, vertex, side, face	 Activities involving observation and composition Focusing on components of shapes Investigating length of sides Focusing on right angles
Grade 3	 Isosceles triangle, equilateral triangle Circle, sphere 	• Angle, center, radius, diameter	 Activities involving observation and composition Focusing on components of shapes Compare length of sides Focusing on shape of angles
Grade 4	 Parallelogram, rhombus, trapezoid Cube, rectangular prism 	• Diagonal line, plane	 Activities involving observation and composition Relationships of straight lines in parallel and perpendicular lines Drawing sketches and nets Expressing a position of an object
Grade 5	 Polygons and regular polygons Prisms and cylinders 	• Base, lateral face	 Activities involving observation and composition Congruence of geometric figures Identifying properties of geometric figures Relationships between circumference of a circle and its diameter (pi) Drawing sketches and nets
Grade 6			 Activities involving observation and composition Reduced and enlarged drawings Symmetric geometric figures (line symmetry and point symmetry)

In grade 1, the main objective is to provide students with enriched experiences that will form the basis for understanding geometric figures. Students are taught to recognize shapes and to grasp the characteristics of shapes.

In grade 2, students need to focus on components of geometric figures and are expected to understand geometric figures such as triangles and quadrilaterals.

In grade 3, students need to focus on components of geometric figures and are expected to understand geometric figures such as isosceles and equilateral triangles.

In grade 4, students need to focus on components of geometric figures and positional relationships. They are expected to understand plane figures such as parallelograms and rhombi, and solid figures such as rectangular prisms.

In grade 5, students are taught how to identify properties of geometric figures. By using those properties, they are expected to deepen their understanding of plane figures and solid figures such as rectangular prisms.

In grade 6, students deepen their understanding of geometric figures through understanding enlarged and reduced drawings and symmetric figures.

(3) Main contents

1. Understanding geometric figures

a. Plane figures

In grade 1, students carry out activities such as observing and composing shapes that are familiar in their everyday life, and they are taught how to recognize and grasp the characteristics of shapes. Students are expected to be able to call shapes by names such as "triangle," "quadrilateral," and "circle," and to be able to study their characteristics.

In grade 2, by focusing on components of geometric figures, students are taught to understand geometric figures such as triangles and quadrilaterals. For example, students should recognize a triangle as a geometric figure surrounded by three straight lines, and a quadrilateral as a geometric figure surrounded by four straight lines. Also, they should recognize a square as a quadrilateral whose four sides are equal in length, and whose four angles are right angles. Other plane figures taught are rectangles and right triangles.

In grade 3, by focusing on components of geometric figures, students are taught to understand geometric figures such as isosceles triangles, equilateral triangles, and circles. An isosceles triangle is a triangle that has two sides the same length. If students fold an isosceles triangle, they find the base angles overlap exactly. By doing this type of activity, students are expected to be able to focus on properties of geometric figures.

In grade 4, students are taught to understand the positional relationships of straight lines such as parallelism and perpendicularity. By understanding these relationships, students can deepen their understanding of previously learned geometric figures such as squares and rectangles. Students are taught about parallelograms, rhombi, and trapezoids, and they are expected to investigate the properties of these geometric figures. Students are also taught the new term, "diagonal."

In grade 5, students are taught about polygons and regular polygons. Students should recognize that equilateral triangles and squares that they learned about

previously can be recognized as instances of regular polygons. They are also expected to understand congruence, where two geometric figures have the same size and shape. Students are taught to find properties of geometric figures; for example, the sum of the measures of the three interior angles of a triangle is 180 degrees, and the sum of the measures of the four interior angles of a quadrilateral is 360 degrees. Also, students should be able to explain the reason for these properties.

In grade 6, students are taught to understand enlarged and reduced drawings and symmetric geometric figures. For example, isosceles triangles and squares, studied previously, have line symmetry, and parallelograms have point symmetry. By being able to look at geometric figures in this way, students are expected to deepen their understanding of plane figures.

b. Solid figures

In grade 1, students do activities such as observation and composition of shapes they find in daily life. They are taught how to recognize and grasp the characteristics of shapes. For example, students are expected to be able to call shapes by "box shape" and "ball shape" and be able to investigate their characteristics.

In grade 2, students are taught about the shape of a box. Also, the following new terms are taught: "vertex," "side," and "face."

In grade 3, a sphere is taught as a solid figure. Students are taught to understand the sphere and its center, radius, and diameter, in relation to the circle.

In grade 4, cubes and rectangular prisms are taught. With respect to rectangular prisms, students are expected to understand the relationships of parallelism and perpendicularity in straight lines and planes. Students are also taught how to draw sketches and nets of rectangular prisms and cubes.

In grade 5, prisms and cylinders are taught. Students are also taught how to draw sketches and nets of prisms and cylinders. The new terms, "base" and "lateral face" are taught.

In grade 6, there is no content about solid figures in the domain "C. Geometric Figures." However, determining volume of prisms and cylinders is taught in the domain "B. Quantities and Measurements."

2. Components of geometric figures

By focusing on components of plane and solid figures, students can deepen their understanding of geometric figures.

In grade 2, students are taught about straight lines, right angles, vertices, sides, and faces. These are components of plane and solid figures. Using these components, students can construct geometric figures and explain their meaning. For example, by using concrete objects such as three sticks, students can make a triangle. By doing this type of activity, it becomes easier to explain that "a geometric figure that is surrounded by three straight lines is a triangle," and it helps students understand the meaning of geometric figures.

In grade 3, by investigating the length of sides that are components of geometric figures, students can perceive geometric figures in such a way that a triangle whose two

sides are equal in length is called an isosceles triangle and a triangle whose three sides are equal in length is called an equilateral triangle. Also, by folding a paper isosceles triangle, students can grasp the property that the sizes of two angles are equal in an isosceles triangle. Here, the focus is also on the component, which is an angle, of a geometric figure. Similarly, in grade 3, the focus is on the following components of a circle: center, diameter, and radius. If students are asked, "Draw a circle whose radius is 5 cm," every student is expected to be able to draw a figure with the same size and shape.

In grade 4, the term "diagonal" is taught, and by focusing on diagonals students are expected to be able to investigate the properties of a parallelogram, rhombus, and trapezoid. For example, with a rhombus, students can find that the two diagonals intersect perpendicularly. Similarly, in grade 4, in conjunction with cubes and rectangular prisms, positional relationships of lines and planes are taught.

In grade 5, the terms, "base" and "lateral face" are taught in relation to prisms and cylinders. By focusing on these components—the base and lateral face—students are able to draw nets of prisms and cylinders.

3. Viewpoint for analyzing geometric figures

a. Observation and Composition

In the content of geometric figures for each grade, activities such as observation and composition are mentioned. Through these activities, students are able to understand the meaning of geometric figures and to find and verify their properties.

In grade 1, students can observe various concrete objects around them, and learn to recognize shapes, grasp their characteristics, and give theses shapes names on their own. Also, students are taught to create concrete objects around them using solids such as blocks or boxes.

In grade 2, students can find objects among the things around them that have the shapes of triangles, quadrilaterals, squares, rectangles or right triangles. Students are taught to do activities where they make geometric figures, for example, by connecting dots in grids, using sticks, folding and cutting paper, and placing colored geometric tiles.

In grade 3, students are taught how to draw isosceles triangles, equilateral triangles, and circles by using a ruler and compass. Also, students are taught about isosceles triangles, equilateral triangles, and circles by observing designs that are tessellated with these types of triangles and designs that are created with circles.

In grade 4, students can categorize quadrilaterals such as parallelograms, rhombi, and trapezoids through observation of common properties, and investigate properties of each individual shape. Also, based on definitions and properties of each geometric figure, students are taught to draw figures using a ruler and compass.

In grade 5, students are taught to deepen their understanding of plane figures through construction of congruent geometric figures while focusing on lengths of sides and sizes of angles. In grade 6, students are expected to enrich their sense of geometric figures by observing and reviewing previously-learned geometric figures from the viewpoint of scaled drawings and symmetry.

Actual activities for composing geometric figures can include: folding or cutting paper; moving, decomposing, or changing geometric figures; and drawing geometric figures using a ruler and compass. These activities are examples of hands-on and experimental activities for learning about geometric figures.

The significance of these activities is as follows:

- These activities help students understand the meaning and terms of geometric figures. For example, by drawing an isosceles triangle, students can understand that two sides are equal in length.
- In problem solving involving geometric figures, students can grasp a problem or have insight to solve a problem. For example, students can find a solution by drawing a circle with a certain radius in a problem where one finds an area where obstacles should not be placed when opening a door.
- Students can discover properties of geometric figures and verify and explain those properties. For example, to find the sum of the interior angles of a quadrilateral, students can think about the problem by breaking the quadrilateral into triangles.
- Students can apply properties of geometric figures in daily life and learning. For example, when measuring the horizontal length of a window frame, using properties of a rectangle, students need to measure only the bottom side.
- Students can acquire knowledge and master and retain skills. For example, when drawing designs with circles using a compass, students become aware that, in a circle, one can cut the circumference into 6 equal-lengths by the radius of the circle, and at the same time, students can acquire skill with the compass.
- Students can acquire ways to observe and analyze geometric figures. For example, by drawing diagonals in rhombuses, students can view the rhombus as a geometric figure that has line symmetry with the diagonals as axes of symmetry.

Composing geometric figures can be a fun activity for students, but it is important that students understand the objectives of the activities, act according to those objectives, and summarize and verify what has been made clear to them.

b. Finding properties of geometric figures and explaining them

In grade 2, students are taught "there are many sizes but all squares have the same shape," and "in a rectangle, opposite sides have equal length."

In grade 3, students are taught that "the sizes of two angles of an isosceles triangle are equal" and "all three angles of an equilateral triangle are equal."

In grade 4, students are taught that "in a parallelogram, lengths of opposite sides and sizes of opposite angles are equal," and in a rhombus, "opposite sides are parallel to each other, and sizes of opposite angles are equal, and the two diagonals intersect perpendicularly and bisect each other." In grade 5, it is important to foster the ability to think logically by finding and explaining properties of triangles and quadrilaterals. For logical thinking methods, there is the inductive method, where various examples are studied and commonality is sought, analogical thinking, where results are inferred from similar cases, and the deductive method, where an explanation is given based on some assumption.

For example, when finding the sum of the measures of the interior angles of a quadrilateral, students can recall the method they used in finding the sum of the measures of the interior angles of a triangle when they investigated by drawing many triangles and wonder if they can use a similar method; or, since the sum of the measures of four angles of a square or a rectangle is 360 degrees, students may think it might also be 360 degrees for a quadrilateral. This is an example of analogical thinking.

The inductive method is where a student draws various triangles, and in measuring the sum of the inner angles of each triangle, finds the sum is 180 degrees.

Based on the fact that the sum of three angles of a triangle is 180 degrees, since a quadrilateral can be divided into two triangles, students can think the sum of the angles of a quadrilateral is 180 multiplied by 2. This is an example of the deductive method.

Fostering logical thinking is done throughout the grades and the domains. In grade 6, students are expected to review previously learned geometric figures from the viewpoints of line symmetry and point symmetry.

c. Drawing sketches and nets

In grade 2, by cutting open a box or constructing a box from an expanded figure, students are taught to understand that a solid figure is composed of plane figures, and to recognize how faces are connected to each other.

In grade 4, when learning about cubes and rectangular prisms through observation of solid figures, students are taught to focus on the positional relationships of vertices, sides, and faces based on a sketch, and to understand the correspondence of vertices, sides, and faces of a solid figure with the components of a net. When teaching this, students should draw sketches and nets, and should become aware of the merits of expressing solid figures in the plane.

In grade 5, while learning about prisms and cylinders using sketches and nets, students deepen their understanding about the properties of solid figures.

d. How to grasp shapes and sizes of geometric figures

In grade 5, congruence is taught through investigating geometric figures from the viewpoints of shape and size.

In grade 6, by investigating geometric figures from the viewpoint of same shapes but different sizes, as with reduced and enlarged drawings, relationships are taught by focusing on the sizes of corresponding angles and the ratios of lengths of corresponding sides.

Also, line symmetry and point symmetry are taught. A figure that has line symmetry can overlap itself completely when it is folded along a straight line. Isosceles triangles, equilateral triangles, squares, rectangles and rhombi are figures with line symmetry. A figure that has point symmetry can overlap itself completely when it is rotated by 180 degrees around a certain point. Squares, rectangles, rhombuses, and parallelograms have point symmetry.

D. Quantitative Relations

(1) Objectives of "D. Quantitative Relations"

The objective of this instructional domain is to acquire mathematical thinking and methods, while understanding and applying the content in the domains, "A. Numbers and Calculations," "B. Quantities and Measurements," and "C. Geometric Figures." This domain also includes the objective of acquiring methods for investigating and expressing quantities and geometric figures. In this revision, ability to think, to make decisions, and to express by using words, numbers, mathematical expressions, diagrams, tables and graphs are emphasized. Therefore, the domain "D. Quantitative Relations" is added from grades 1 and 2 to enhance these abilities.

In this domain, the main contents are "ideas of functions," "mathematical expressions and their interpretation," and "sorting and interpreting data." In each area, it is important to enable students to express and investigate quantities and their relations with numbers, mathematical expressions, diagrams, tables, and graphs, and to express and investigate them with words, reasoning, and explanations. Especially in grades 1 and 2, the contents, "mathematical expressions and interpreting them," and "sorting data and interpreting them" (which were previously located in "A. Numbers and Calculations") are moved to "D. Quantitative relations" in order to improve the organization and enrichment of the content.

The idea of functions describes a method whereby a problem is solved by focusing on patterns of change or correspondence involving quantities or geometric figures. It is especially important to investigate the relationship between two quantities that change simultaneously, and interpret and express characteristics and tendencies of the relationship.

Mathematical expressions are sometimes called the language of mathematics, and they play important roles in expressing objects and their relationships accurately and concisely so they can be understood. It is important to be able to interpret mathematical expressions and to use them in relationship to words and diagrams.

In sorting and interpreting data, it is important to be able to collect and sort data according to one's objectives, and to express the data clearly by using methods such as tables and graphs, and to investigate their characteristics and interpret them. Also, it is important to be able to choose graphs and tables appropriate to the objective, to use them with each other, and to understand how to read and apply them.

(2) Overview of the contents of "D. Quantitative Relations"

The table below shows the main contents in each grade categorized by "Ideas of functions," "Mathematical expressions and interpreting them," and "Sorting and interpreting data."

Grade	Ideas of functions	Mathematical expressions and their interpretation	Sorting and interpreting data
Grade 1	 Correspondence between two objects Sizes of numbers (large/small), order of numbers Seeing a number as sum or difference of other numbers 	• Mathematical expressions of addition and subtraction and how to interpret them	• Expressing and interpreting numbers of objects using pictures and diagrams
Grade 2	 Sizes of numbers (large/small), order of numbers Seeing a number as the product of other numbers How a product changes when the multiplier increases by 1. 	 Relationship between addition and subtraction Mathematical expressions of multiplication and how to interpret them Mathematical expressions with () and symbols like 	• Sort quantities in daily life and express them in simple tables or graphs, and interpret them
Grade 3	• Including a case in which the multiplicand or multiplier is zero, how a product changes when the multiplier increases or decreases by 1	 Mathematical expressions of division and how to interpret them Expressing quantitative relations in mathematical expressions, relating mathematical expressions to diagrams Mathematical expressions with symbols like 	 Sort and express data clearly in tables or graphs, and interpret them Interpret bar graphs and construct them
Grade 4	• Relations between two quantities and broken-line graphs	 Mathematical expressions that have a combination of the four arithmetic operations and (). Idea of formulas and applying them Mathematical expressions with symbols like □ and △ Summary of properties of four arithmetic operations 	 Sort data from two viewpoints and investigate them Interpret and construct broken-line graphs
Grade 5	• Simple case of proportional relationship	• Mathematical expressions to express relationships between quantities	 Percentage Sorting and organizing data Pie chart and percentage bar graph
Grade 6	 Ratios Investigate proportional relations by using mathematical expressions, graphs, and tables Solving problems using proportional relationships Inverse proportion 	• Mathematical expressions with letters such as <i>a</i> , <i>x</i>	 The mean of data Tables and graphs that express frequency distribution Investigate possible outcomes

In grade 1, mathematical expressions of addition and subtraction and their interpretation are taught. Also, students are taught to represent numbers of objects by using pictures and diagrams and to be able to interpret such representations.

In grade 2, students are taught about expressions using the mutual relationship between addition and subtraction, and mathematical expressions of multiplication and their interpretation. Students are also taught to organize numbers and quantities from their everyday lives, represent them by using simple tables and graphs, and interpret such representations.

In grade 3, mathematical expressions of division and how to interpret them are taught, as well as how to find relationships between mathematical expressions that represent relationships of quantities and diagrams, and how to represent relationships with mathematical expressions that use symbols such as \Box . Also, expressing phenomena using tables and bar graphs, and how to interpret tables and bar graphs, are taught.

In grade 4, students are taught to represent the relationship between two quantities that change simultaneously on broken-line graphs and to interpret their characteristics. Also, students are taught to understand mathematical expressions that represent quantitative relationships, how they can use these expressions, and mathematical expressions that use \Box and \triangle . Students are taught to gather, sort, and organize data according to their objectives, to clearly represent data in tables and broken-line graphs, and to investigate features of data.

In grade 5, students are taught to investigate simple cases of proportional relationships by using tables. Also, students are taught about the relationship between two quantities expressed in a simple mathematical expression. Students are also expected to understand percentage, and to gather, sort, and organize data according to their objectives, to represent data in pie charts and percentage in bar graphs, and to investigate the features of this data.

In grade 6, students are taught to investigate the features of ratios and proportional relationships by using mathematical expressions, tables, and graphs; to solve a problem by using proportional relationships; and to become aware of inversely proportional relationships. Also, they are taught about mathematical expressions that use letters such as "*a*" and "*x*." They are also taught about the mean of data, frequency distribution, and systematic analysis of all possible outcomes.

(3) Explanation of Main contents

1. Ideas of functions

A problem can be solved by giving consideration to patterns of change and correspondence involving quantities and geometric figures. By using ideas of functions, it is important that students will deepen their understanding of quantities and geometric figures using these ideas, investigate two simultaneously changing quantities, and interpret their features and tendencies.

To make the best use of the concept of functions, it is important to consider the following:

First, one must focus on how an issue regarding quantities and geometric figures in a certain situations relates to other issues. For example, when one quantity changes, does the other quantity change? Or if one quantity is determined, does it mean the other quantity is also determined? If some components of a geometric figure are determined, does it mean the other components or matters are determined? By focusing on these

relationships, students are able to investigate any dependency relationship between two quantities. This is the first step in forming ideas of functions. In considering this, it is also important to clarify the range of a matter one is investigating.

Secondly, it is necessary to investigate the characteristic of change and correspondence between two matters. Relationships such as regularity can be found between two simultaneously changing quantities. By expressing quantities and their relationships in words, numbers, mathematical expressions, diagrams, tables, and graphs, students can interpret situations of regularity in change and correspondence more specifically.

Thirdly, the recognition of regularity in change and correspondence obtained in the process above needs to be used for solving problems, and for enabling students to express and explain the steps in their thinking, and their results.

In grades 1 through 3, students are taught to make a correspondence between two objects, to represent a number as sum or difference of two numbers, to represent a number as product of two numbers, and to think about how a product changes when a multiplier increases by 1. These contents relate to the contents in "A. Numbers and Calculations," and exhibit the ideas of functions used here.

In grade 4, students will discover the relationship between two simultaneously changing quantities from the events in daily life, and they will express this relationship in tables and broken-line graphs, while interpreting their features.

In grade 5, simple cases of proportional relationships are investigated by using tables to describe relationships between two simultaneously changing quantities.

In grade 6, students are taught to understand ratios, and they are also taught to investigate proportional relationships as one type of relationship between two simultaneously changing quantities using mathematical expressions, tables and graphs. They are taught to solve problems using proportional relationships; inversely proportional relationships are also taught in this grade.

2. Mathematical expressions and their interpretation

There are words, numbers, mathematical expressions, diagrams, tables and graphs that work as methods for representing quantities and their relationships in daily life. Among them, mathematical expressions are excellent for representing phenomena and relationships concisely, clearly, accurately, and in general terms. In teaching mathematical expressions, students are able to represent concrete matters and relationships in mathematical expressions by associating them with concrete situations. Furthermore, it is important that students become able to interpret situations through mathematical expressions, and to represent them by using words or diagrams, and to process and promote thinking by using mathematical expressions. It is also important that students become able to explain their ideas by relating mathematical expressions to words, diagrams, tables, and graphs, and to clearly communicate these ideas to one another.

Of mathematical expressions, there are those such as 2+3, $\Box \times 5$, x-5, and those that contain an equal sign, such as 2+3=5, $\Box \times 3=12$, $a \times b = b \times a$. There are also mathematical expressions that contain words such as "unit cost × number of items =

total cost." In the first group, it is important for students to be able to see that the expression 2+3 represents a quantity in a certain situation. Also, students should see that mathematical expressions with equal signs such as \Box +3 = \triangle , *x*+3=8, represent relationships between quantities in certain situations.

Additionally, in mathematical expressions, by using (), one can represent more than one phenomena in a single expression (forming a compound expression).

The following are some uses of mathematical expressions:

- (a) to express things and relations simply, explicitly, and generally;
- (b) to be able to be manipulated formally apart from the concrete meaning they are expressing;
- (c) to be able to interpret a concrete situation and to interpret relationships, and to be able to examine them more accurately;
- (d) to express the way one thinks, and to communicate with other people.

Interpreting mathematical expressions involves the following:

- (a) interpreting a concrete situation from the expression;
- (b) generalizing the phenomena or the relations that the expression represents;
- (c) widening the range of numbers in an expression (e.g., expanding form whole numbers to decimal numbers), that is, to interpret it broadly;
- (d) understanding the thinking process behind problem solving, from a mathematical expression;
- (e) interpreting mathematical expressions in correspondence with a number line.

In grade 1, students are taught to represent situations where addition and subtraction are used in mathematical expressions and to interpret these expressions. For example, if the situation is described as "3 people were playing and 4 people just came to join them," students are taught to write 3+4 to represent the situation. But, since concrete numbers are used in grades 1 and 2 and calculation immediately leads to one number, students rarely become aware of the fact that 3+4 represents a concrete phenomenon. Therefore, it is important to teach students to focus on the meaning of mathematical expressions instead of paying attention solely to getting results.

Also, in grade 2, students are taught to explain the mutual relationship between addition and subtraction, and to be able to represent multiplicative situations with mathematical expressions. In this revision, it was decided to teach mathematical expressions with symbols such as \Box . In grade 2, this is mentioned in "Remarks Concerning Content"-(2).

In grade 3, students are taught to represent division situations with mathematical expressions, and to interpret them. They are also taught to associate mathematical expressions that represent quantitative relations with diagrams, and to represent a mathematical expression with a symbol such as \Box . For example, an answer to 12÷3 can be written as \Box in 3 × \Box = 12.

In grade 4, students are taught to deepen their understanding of mathematical expressions that represent quantitative relations. They are also taught to construct mathematical expressions using $\Box \rightarrow and \bigtriangleup$ acting as letters.

In grade 5, students are taught about relationships of quantities that are represented by simple mathematical expressions.

In grade 6, students are taught to represent mathematical expressions with letters such as "a" and "x".

3. Sorting and interpreting data

The main objectives here are to collect, sort and organize data according to one's purpose, to represent it clearly in tables and graphs, and to investigate and interpret the characteristics of the data. Through these activities, it is also important to foster thinking for making good judgments and rational predictions. This is especially important in modern society, which is flooded with much information. If students can see how mathematics can be applied in this way, they can appreciate its value.

To enhance students' ability to collect, sort, organize, represent, and interpret data, it is important that students acquire knowledge and skills, and learn ways of thinking and representation, and learn ways of interpreting data through the following activities:

- a. Define problems and objectives, gather the necessary data, and quantify them appropriately.
- b. Sort and organize data by making tables and graphs, and find the mean and percentage in order to understand the characteristics and tendencies of the data.
- c. By focusing on the characteristics and tendencies of data, make a prediction and judgment about the situation, apply it to problem solving, and express and explain the thinking process and results.

In order for students to learn to sort, organize, and interpret data, students in grade 1 learn about the quantity of objects and compare numbers by using concrete objects. Also, they are taught to represent situations by using pictures and diagrams and then they are taught to interpret them.

In grade 2, they are taught to sort and organize quantities surrounding them. They are taught to represent them by using simple tables and graphs and they are taught to interpret tables and graphs.

In grade 3, they are taught to sort and organize quantities. They are taught to represent them clearly by using tables and bar graphs and they are taught to interpret tables and graphs.

In grade 4, they are taught to collect, sort, and organize quantities according to an objective. They are taught to represent the quantities clearly using tables and broken-line graphs, and they are taught to investigate the characteristics of the data.

In grade 5, focusing on ratio in the sense of a percentage of the whole, students are taught to find characteristics of data using pie charts and percentage bar graphs. It is important for the students to utilize the methods for analyzing data by choosing the manner of representation—table, bar graph, broken-line graph, pie chart, or percentage bar graph—according to their objective; representing the data by in accordance with what they see in the tables and graphs; interpreting characteristics of the data; and making judgments.

In grade 6, students are taught about the mean and distribution of data. They are also taught how to represent data by using frequency tables and graphs, and about statistical investigation. Finally, regarding concrete phenomena, students are taught how to investigate possible outcomes by organizing them.

Chapter 3. Content by Grades

1. The Content of Grade 1

A. Numbers and Calculations

A (1) The meaning of numbers and their representation

- (1) Through activities such as the counting of objects, students will understand the meaning of numbers and be able to use numbers appropriately.
 - a. To compare numbers of objects in groups by making one-to-one correspondence between objects.
 - b. To correctly count or represent the number and order of objects.
 - c. To develop a sequence of numbers and to represent numbers on a number line by thinking about the order and the size of those numbers.
 - d. To consider a number in relation to other numbers in ways such as regarding it as a sum or difference of other numbers.
 - e. To understand how to represent 2-digit numbers.
 - f. To understand how to represent 3-digit numbers in simple cases.
 - g. To consider numbers using ten as a unit.

Mathematical Activities (1)

a. Activities such as counting concrete objects as a group or dividing them equally, or sorting and representing them.

Terms/Symbols

ones place, tens place

Students are first taught to compare numbers of concrete objects through one-to-one correspondence. Then they are taught to correctly count those objects and to represent them by using numerals. Through these activities, students learn the size and order of numbers, and gradually learn the meaning and the system of numbers through 100.

Also, students learn how to represent 3-digit numbers in simple cases.

a. Comparing numbers

When comparing the number of objects, one does not have to count them—one can judge the size of a number by making one-to-one correspondence. For example, in the diagram below, when you find a one-to-one correspondence between blocks **■** and marbles •, you can understand that the number of marbles is greater than that of blocks.



When comparing objects that are difficult to compare directly, you can replace them with concrete objects. For example, when you count the number of sounds, the number of passing cars, or trees in the school yard—something you cannot manipulate by hand—you can replace the objects with something easy to count, e.g. making these objects correspond to marbles and then counting the number of the marbles.

In order to help students learn to count correctly by one-to-one correspondence between objects and number words, or to compare using numbers, it is important to use activities where correspondence between objects is made in this way.

b. Counting the number and order of objects

When counting the number of objects, it is important to understand what is being counted as a group. Then, make a one-to-one correspondence between what is being counted and the number words, "one, two, three, four, …," reciting the number words in order, then represent the number of the objects by the last number word.

Also, in expressing order of objects, it is important to foster an understanding of numbers. When examining objects in order, if you make correspondences between the objects and the numbers, the numbers can be used to represent their positions. In this case, you will know that the number used to represent the last position is the same as the total number of the objects.

In the case of 0, it is necessary to help students gradually understand the following ideas.

- 1) Through experiences such as not having any score in a "point game," or in which the number of concrete objects decreases one by one until nothing is left, students can understand that 0 stands for nothing. It is important that students perceive 0 as a number just like any other number.
- 2) 0 represents a vacant place in the base-10 numeration system, such as the ones place of 70 or 80.
- 3) 0 represents the origin on a number line.

c. Size and order of numbers and the number line

A *number line* represents numbers by the distances of points on the line from a reference point, to which 0 has been corresponded, using a selected length as the unit. By using this number line, the size, order, and sequence of numbers can be represented more clearly.

It is good to introduce a number line in relation to the order of objects in a line. Also, students should gradually learn the number line with intervals other than 1, e.g., 5 or 10, or a number line starting at a number other than 0, e.g., 50.

The term "number line" is introduced in grade 3.

d. Considering a number as a sum or difference of other numbers.

The starting point for understanding whole numbers is in the process of counting objects. Soon, through activities, students start to grasp a number constitutively by composing and decomposing it. The understanding of composition and decomposition of numbers is indispensable for establishing the meaning of numbers. For example, as in the drawing below, students start to perceive 5 marbles as a combination of parts.



Students also learn to see a number in relation to other numbers. For example, if you see 8 in relation to 10, since 8 is smaller than 10 by 2, you can use the expression 10 - 2.

This way of viewing numbers is also important because it forms the foundational knowledge for understanding regrouping with addition and subtraction. For example, in the case of 8+6, you may think: "first, 8 is less than 10. 6 is the sum of 2 and 4, and 8 and 2 make 10. This 10 and 4 make 14."

So it is important to be able to see numbers from various perspectives, thus enriching one's sense of numbers.

e. How to represent 2-digit numbers

In grade 1, the objective is to help students have a basic understanding of the principle of the base-10 numeration system.

It is important to introduce 2-digit numbers through activities such as counting concrete objects or representing numbers using concrete objects. Through these activities, students can understand that two-digit numbers are based on the idea of groups of ten and a remainder. This understanding enriches a sense of how numbers are composed.

When representing numbers using the base-10 numeration system, because different places are used to represent the size of units such as one, ten, and hundred, only a finite numbers of symbols are necessary. In this system, judging the size of numbers or carrying out the four arithmetic operations with a computational algorithm (introduced from grade 2) becomes easy. It is important to help students understand the merit of this system as they study specific contents of numbers and calculations.

To foster an understanding of the base-10 numeration system, the terms "ones place" and "tens place" are introduced. For example, for number 43, its ones place is 3 and its tens place is 4. This means there are 3 ones and 4 tens.

f. How to represent 3-digit numbers in simple cases

Students learn how to represent 3-digit numbers in simple cases. Simple cases mean numbers up to about 120.

Through activities with concrete objects such as making groups of 100, 10 and a remainder, students may solidify their understanding of the meaning and representation of 2-digit numbers. Moreover, these activities will lead smoothly to the study of 3-digit numbers taught in grade 2.

g. Considering numbers using ten as a unit.

In grade 1, students learn how to represent numbers using the base-10 numeration system through such activities as making groups of ten. In grade 1, students learn to see the size of numbers using ten as a unit.

Here, thinking about numbers using ten as a unit means to see the group of tens in numbers such as "40 is four tens", or to see numbers as several tens such as "Six tens makes sixty."

By guiding students to see numbers in units of ten, you deepen their understanding of the composition of numbers, and connect them to a method of adding and subtracting where numbers are seen as units of ten.

Mathematical Activities (1) a

Counting concrete objects by categorizing or making equal sized groups, and representing the results of counting in and organized manner.

The objectives of this activity are to help students acquire the fundamental ability to count, to understand the basic idea of the base-10 numeration system, and to enrich their sense of numbers.

When counting concrete objects, activities such as counting by groups of 2, 5, and 10, or sorting and representing the concrete objects that students counted on a chart are used. Counting objects by making groups of 10 especially becomes the foundation for understanding how the base-10 numeration system works. When teaching this, it is important to have students make a suitable group of numbers, count the objects using that group, and sort and represent what they counted, instead of counting pre-grouped objects. Also, to develop the process of counting, it is important to let students count objects in their everyday life, not only objects on the desk.

In cases where you want to equally divide concrete objects, use activities such as dividing the entire group into several groups of equal numbers or dividing the entire group into different numbers of equal groups. For example, explain how to divide 8 pencils by groups of twos or groups of fours. Explain how many people can get pencils if one divides the pencils equally by manipulating them and using charts, or sorting and representing the results of division using mathematical expressions. Through these activities, students see the number 8 from various perspectives and enrich their sense of numbers.

A (2) Addition and subtraction

- (2) Students will understand the meaning of addition and subtraction, and will be able to use those operations appropriately.
 - a. To become aware of situations in which addition and subtraction may be applied.
 - b. To investigate the ways to calculate the sum of two 1-digit numbers and the difference, which is the inverse, so that the students can add and subtract accurately and reliably.
 - c. To think about the ways to calculate simple cases of sums and differences involving 2-digit numbers.

Mathematical Activities (1)

b. Representing the meaning of mathematical operations and the processes of calculation by using concrete objects, words, mathematical expressions, and diagrams.

Terms/Symbols

+, -

Students are expected to understand when to apply addition and subtraction to concrete phenomena as well as the meaning of addition and subtraction. Also, students are expected to think about how to carry out addition of a one-digit number with another one-digit number, and the inverse calculation^{*}, to understand the meaning of these calculations, to think about and explain how to carry out these operations for themselves, to actually carry out these calculations, and to utilize these calculations in their daily life. Furthermore, students are expected to think about how to carry out addition and subtraction of 2-digit numbers in simple cases and to be able to explain them.

It is important that the calculation activities like "understanding the meaning of calculation," "thinking about how to carry out calculations," and "mastering calculations and appropriately applying them" are done in every grade.

a. Cases where addition and subtraction are used, and their meanings

The following list provides examples of cases where addition and subtraction are used.

^{*} {Translators' note: i.e. subtraction, including (2-digit) – (1-digit) = (1-digit).]

1) Cases where addition is used.

- (A) To obtain the result of adding something to an original number or amount, or to increment a certain amount from the original number or amount. (Increase)
- (B) To find the total of two co-existing numbers or quantities. (Union)
- (C) To find the number or order that is a certain number of places following another number or order. (Addition involving ordinal numbers)

2) Cases where subtraction is used.

- (A) To find the result of removing or decrementing certain numbers or quantities from the original numbers or quantities. (Take away)
- (B) To find the difference of two numbers or quantities. (Comparison)
- (C) To determine the number or order that is a certain number of places preceding another number or order, or to find the difference of the two. (Subtraction involving ordinal numbers)

To teach these ideas, it is important to use concrete situations and to help students understand that in all of these cases the same addition or subtraction can be used. Therefore, gradually generalizing cases where addition and subtraction are used helps students understand the meaning concretely. And it is important to help students understand through such activities as concrete manipulation that addition is the calculation for finding the number of elements of two sets combined and that subtraction is the calculation for finding the number of elements of one group when the original group is separated into two.

b. Addition of 1-digit numbers and its inverse, subtraction

For addition of a one-digit number with another one-digit number, and subtraction as the inverse of this addition, there are two cases: one is an addition whose sum is smaller than or equal to 10, and the subtraction that is the inverse of this addition. The other is an addition whose sum is greater than 10, and the subtraction that is the inverse of this addition.

You can help students understand the former type of calculation through activities with concrete objects. For the latter, help students think about this as "10 and more" through activities with concrete objects, and they will be able to explain how to do this calculation logically. In either case, this will become the basis for addition and subtraction taught later, so it is important to teach it using such means as concrete objects so that students will understand the meaning of calculations, think about how to carry out these calculations, and become able to carry out these calculations without problems.

Especially in the case of subtraction, there are many possible ways to perform calculations. The principle methods are minuend decomposition and subtrahend decomposition. For example, in the case of 12 - 7, minuend decomposition subtracts 7 from 10 and adds 2, as in (10 - 7) + 2. Subtrahend decomposition subtracts numbers in steps, as in (12 - 2) - 5. If you introduce activities with blocks, etc., the difference would be between taking away from a group of ten, or taking away from the ungrouped objects

first. The choice of these two methods should be flexible, depending on the size of the numbers, but it is important to instruct students according to their level.

c. Simple cases of addition and subtraction involving 2-digit numbers

Based on additions and subtractions of a one-digit number and another one digit number and what has been learned so far about numbers, students are supposed to think about how to perform the addition and subtraction of two-digit numbers in simple cases.

Simple cases are as follows:

1) Addition and subtraction of numbers using ten as a unit

Addition and subtraction of numbers using ten as a unit means, for example, calculations like 20+40 or 70-30. These calculations can be thought of as 2+4 and 7-3, when related to the idea of seeing numbers using ten as a unit.

Calculations whose sum exceeds 100 will be taught in grade 2.

2) Addition and subtraction of 1 and 2-digit numbers without regrouping

Addition and subtraction of 2- and 1-digit numbers without regrouping here means, for example, addition like 13+4 or 20+5 and subtraction like 15–2 or 38–8.

In simple cases such as the above, we want to strengthen students' understanding of calculations of 1-digit and 2-digit numbers by teaching addition and subtraction including 2-digit numbers. Understanding how to carry out these calculations will be of help in grade 2 when they formally learn addition and subtraction of 2-digit numbers, subtraction being the inverse of the addition.

Mathematical Activities (1) b

Representing the meaning of mathematical operations and the processes of calculation by using concrete objects, words, mathematical expressions, and diagrams

The objective of this activity is to help students understand the meaning of calculations based on concrete situations, to come up with new ways of calculating on their own, and to use and express calculation methods they have already learned.

For example, for a problem like, "Taro picked 8 acorns. Hanako picked 7 acorns. How many acorns do they have in total?" ask students to think about the meaning of the calculation and ways of adding with regrouping.

From this problem students can see that this is a case of finding the total of two coexisting numbers or quantities, which can be represented using the mathematical expression 8+7. Also, based on the meaning of the calculation in this case, we can think about the manner of calculation, as in the following example. 1) Decompose the 7 acorns Hanako picked.



Decompose 7 into 2 and 5. Adding 2 to 8 makes 10. Adding 10 and 5 makes 15.

2) Decompose the 8 acorns Taro picked.



Decompose 8 into 5 and 3. Adding 3 and 7 makes 10. Adding 10 and 5 makes 15.

Thus, we can think of this problem as (8+2)+5 by decomposing the addend, 7, or think of it as 5+(3+7) by decomposing the other addend, 8. This will help students realize that 10 is made in both cases, so that students can find out how to add without carrying over.

B. Quantities and Measurements

B (1) The basis for understanding quantities and measurements

- (1) Students will enhance their experiences that will form the foundation of understanding quantities and their measurements through activities such as comparison of quantities.
 - a. To compare length, area, and volume.
 - b. To compare quantities by using familiar objects as units.

Mathematical Activities (1)

c. Comparing objects from the surroundings in terms of their lengths, areas, and volumes either directly or by using another objects as a reference.

The objective of grade 1 is to provide enriching experiences that will form the basis for understanding the meaning of quantities such as length, area, and volume, and the meaning of measurement before actually measuring using units from their everyday life.

In grades 1 and 2, during size comparison activities, area may be called "the size of the space" and volume may be called "capacity."

The important thing about teaching the meaning of a unit of quantity and its measurement is to deepen the understanding of a quantity through activities in which the size of an object is compared directly or indirectly. Furthermore, it is necessary to have students decide on a unit to use and to understand that they can represent the size of the whole by counting the number of units. Through these activities, have students gradually understand that a certain amount can be divided into two or more parts without changing the amount as a whole, or that the original size can be reconstituted by adding up these parts.

a. Direct comparison of quantities

When comparing the size of objects, sometimes one can move objects, juxtapose one object with another to compare their sizes directly. For example, when you want to compare the length of two pencils, you can compare the length by moving the pencils and placing them side-by-side or superimposing one onto the other. In this case, since it is important to align the reference points, be sure to align one end of each pencil, and judge which one is longer by comparing the other ends of the pencils. As for comparing the area, when you can superimpose one object onto the other, if one is completely covered by the other, you can see which is larger. Volume is more difficult to compare than length or area. But, for example, if you can put a small box into a larger one, you can directly compare the volume of two boxes.



b. Comparing quantities using arbitrary units

When comparing the size of objects, sometimes it is difficult to move the objects in order to compare them directly. In this case, you can compare the size by using some another object as a reference. Imagine comparing the length and width of a desk. You may want to use a string or a stick to copy those lengths since the length and width of a desk cannot be moved around. By copying the length and width of a desk, you can compare these two lengths based on what was discussed earlier. When an appropriate object cannot be found, one can use the length of an object such as a new pencil or a new eraser as a unit. We can then represent the length of each object by how many units will be required to make the same length, and compare the resulting numbers to compare the lengths of objects.

As for area, even when you cannot superimpose one object onto the other, you can still compare area by covering the objects with color tiles and counting the number of those color tiles. It is important to make students aware of the meaning of area through such activities as covering with color tiles or shading individual squares on grid paper. Comparison of the volume of two objects can be done by filling one container with water and moving the water into another container, by filling two containers with water and moving water into the third container, or by counting how many glasses of water are in the container.

Thus, by expressing the size of objects numerically using everyday items as units, you can compare three or more objects efficiently. Moreover, we can say one object is so much larger than the other or one is so many times as large as the other.

Mathematical Activities (1) c

Comparing objects from the surroundings in terms of their lengths, areas, and volumes either directly or by using another objects as a reference.

The objective of this activity is for students to be able to utilize the units of area and volume and the idea of measurement of area and volume in their daily lives.

In this activity, students actually compare the length, area, or volume of objects at school and in their homes. For example, when comparing the area of two blankets, students may place one over the other (direct comparison). Or, when comparing the volume of water in bottles, students may fill each bottle with water and pour the water into tanks (indirect comparison). Or, when comparing the width and the length of a desk, students may represent each length using erasers and see how many erasers long the width and length of the desk are (measurement with arbitrary units).

Through comparisons and measurements in various situations, students can understand what it means that the length is about how long an object extends, the area is about how spacious an object is, and the volume is about how much an object can contain. Thus, they will understand these attributes and how to measure them; moreover, these comparisons will help them acquire a sense of the size of quantities.

B (2) Reading time

(2) Students will be able to read clock times in their daily lives.

In the everyday life of a student, it becomes necessary to be able to tell time from relatively early on. In grade 1, students are taught how to tell time (hour, minute) from a clock with long and short hands. Also, it is important to help students become interested in time by relating time to activities in their daily life.

C. Geometric Figures

C (1) Basic understanding of geometric figures

- (2) Students will enrich their experiences that form the foundation for understanding the concepts of geometric figures through observing the shapes of objects from students' everyday lives and composing shapes using such objects.
 - a. To recognize the shapes of objects or to notice their features.
 - b. To represent the position of an object by correctly using such words concerning directions and positions as "front and rear," "right and left," and "above and below."

Mathematical Activities (1)

d. Identifying various geometric figures from their surroundings, and composing and decomposing geometric figures using concrete objects.

The objective of grade 1 is to provide rich experiences that will become the basis for understanding plane figures and solid figures.

Students have experienced such everyday activities as piling up and tiling blocks and boxes, or folding, superimposing, and comparing pieces of paper while they were playing in their life before entering school. Drawing on their experience, it is important to teach students to use shapes around them as objects of observation or composition, to find those shapes around them, actually pick them up, and create different shapes with them. Through these activities, students are gradually able to recognize those shapes regardless of their color, size, place or material, and understand their characteristics.

Activities where students observe and compose shapes are emphasized. At the same time, students will be taught to describe in words how they compose and decompose those shapes.

a. Shapes and ways to recognize their features

"To recognize the shapes of objects" means to be able to identify triangular, rectangular and circular shapes in such everyday concrete objects as floor tiles or tiled cobblestones by disregarding properties other than the shape. It also means the ability to find triangular, rectangular or circular shapes as the constituent parts of everyday solids such as boxes, tubes, and balls.

"To recognize the features [of shapes]" means to be able to recognize the properties of objects. For example: "triangles" and "rectangles" have corners but "circles" do not; the corners of "triangles" are sharp; the numbers of corners of "a triangle" and "a rectangle" are different when comparing these two. It also means recognizing functional aspects of solids; for example: boxes have flat faces but the balls do not; a tube can roll over easily or be piled up depending on how they are placed; balls have a shape that rolls easily; or boxes have shapes that can be piled up. To teach students to recognize the shapes of objects or to see the properties of shapes, it is important to have them do such activities as composing shapes in their daily life using solid figures such as blocks or boxes, or imagining the original shape from the shapes they constructed. Activities like presenting a shape and having students collect concrete objects with similar shapes, or asking students where they can find those shapes or what properties those shapes have, are also important. Activities like observing a box and copying its face on paper, and having students find those shapes in their surroundings, are also important.

Introducing these activities and gradually emphasizing the process of discovering the properties and nature of shapes will enrich students' experiences and will provide the basis for understanding plane and solid figures.

b. Direction and Position

Observe how students express the direction and position of objects in their daily life, and teach them how to express the position of objects by using such words as "front" and "back," "left" and "right," and "top" and "bottom" correctly.

For example, have students be able to express two spatial relationships in the classroom, such as: "the clock on the wall is above the class schedule" or "the TV is to the left of the blackboard from my point of view." In this study, make sure students really understand directions and positions through such activities as lining in single file or aligning concrete objects.

Mathematical Activities (1) d

Identifying various geometric figures from their surroundings, and composing and decomposing geometric figures using concrete objects

The objective of this activity is to raise students' awareness of and interest in the shapes of objects, and to have them accumulate rich experiences that will form the foundation for the study of shapes.

Have students observe concrete objects such as boxes, tea canisters, or blocks and find the "rectangle" in boxes, the "circle" in tea canisters, and the "triangle" in blocks. They can also copy those shapes on paper and cut them out to make those shapes.

Students can do activities such as making the rocket shapes or house shapes shown at right by arranging shapes or color tiles they have made.

Students can make squares or triangles using sticks, etc.

Students can cut an origami paper into halves to make "triangles." Also, they can make a "rectangles" by combining two "triangles."

These activities not only provide students with rich experiences that will provide the basis for understanding shapes, but also enrich their sense of shapes by familiarizing students with shapes by relating them to their daily lives. When arranging color tiles and making shapes, it is possible to introduce such activities as sliding, turning, and flipping color tiles.





D. Quantitative Relations

D (1) The mathematical expressions of addition and subtraction

(1) Students will be able to represent the cases in which addition and subtraction may be applied by using mathematical expressions and will be able to interpret such expressions.

Mathematical Activities (1)

e. Writing mathematical expressions to represent quantities in concrete situations and interpreting mathematical expressions in context.

Terms/Symbols

+, -, =

It is important to emphasize expressing the concrete situations in which addition or subtraction is applied using mathematical expressions with + or – symbols, interpreting those expressions in a concrete context, or interpreting mathematical expressions and representing them with diagrams or concrete objects.

Mathematical expressions are thought to represent situations or processes for finding answers, and are important as expressions unique to mathematics.

To interpret a mathematical expression means that, looking at a mathematical expression, students relate it to a corresponding concrete situation, and understand the relationship between quantities in that situation. Students should become able to express mathematical situations and quantities using words, diagrams, and concrete objects.

You can create activities for interpreting mathematical expressions in relation to concrete situations like the following example: Based on the mathematical expression 5+3=8, students might create a story like, "There are 5 students playing in a sandbox. Three more students come. Now, there are 8 students playing." Or, from the expression 5+3, students might create problems such as, "There are 5 students playing in a sandbox. Three more students come. How many students are playing now?" In this way, students can invent concrete situations using words, diagrams, and concrete objects for given mathematical expressions.

Mathematical Activities (1) e

Writing mathematical expressions to represent quantities in concrete situations and interpreting mathematical expressions in context

The objective of this activity is to be able to interpret mathematical expressions with addition and subtraction, or to utilize those expressions to represent concrete situations, or to relate mathematical expressions to concrete situations. To promote activities relating mathematical expressions of addition and subtraction with concrete objects in the classroom or in school or real situations in daily life, students are taught to find situations that can be expressed by those expressions and express them using words, pictures, and diagrams, and to represent the quantities found in real life using mathematical expressions, or to make mathematical problems.

For example, a student can find the total number of morning glory seeds collected yesterday and today using addition. Or, from the expression 8–3=5, a student can tell a story like, "There are 8 students playing in a sandbox. Three students leave. Now there are 5 students playing." Or, from the expression 6-3+7, a student can make a mathematical problem like, "There are 6 squirrels. Three of them leave. But another 7 come. How many squirrels are there now?" and represent the situation using pictures.

To foster a deeper understanding of mathematical expressions and to relate them with concrete situations through this kind of teaching is important in the study of the four arithmetic operations in the following grades.

D (2) Representation of quantities using pictures and diagrams

(2) Students will be able to represent numbers of objects by using pictures and diagrams and will be able to interpret such representations.

When counting a collection of objects that consist of different kinds of object—for example, ducks, cats, rabbits, and squirrels—students are expected to represent the objects using pictures or diagrams like the one below:



Also, students are expected to interpret the characteristics of the collection, such as where there are the most objects or fewest objects, from the pictures or diagrams.

Mathematical Activities

(1) For the contents of "A: Numbers and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.

- a. Counting concrete objects by categorizing or making equal size groups, and representing the results of counting in an organized manner.
- b. Representing the meaning of mathematical operations and the processes of calculations by using concrete objects, words, mathematical expressions, and diagrams.
- c. Comparing objects from the surroundings in terms of their lengths, areas, and volumes either directly or by using another object as a reference.
- d. Identifying various geometric figures from their surroundings, and composing and decomposing geometric figures using concrete objects.
- e. Writing mathematical expressions to represent quantities in concrete situations and interpreting mathematical expressions in context.

Terms/Symbols

ones place, tens place, +, -, =

2. The Content of Grade 2

A. Numbers and Calculations

A (1) The meaning of numbers and their representation

- (1) Students will extend their abilities to use numbers by fostering an understanding of the meaning and representation of numbers.
 - a. To count objects by rearranging them into groups of the same size or by classifying them.
 - b. To understand the base-10 numeration system so that they can write and compare numbers up to 4-digit numbers.
 - c. To understand relative size of numbers by regarding 10 or 100 as a unit.
 - d. To consider a number in relation to other numbers, such as regarding it as a product of other numbers.
 - e. To become aware of simple fractions such as ½ and ¼.

Mathematical Activities (1)

a. Identifying situations in everyday life where whole numbers are being used.

Terms/Symbols

>,<

<u>Remarks Concerning Content</u> (1) As for the content A-(1), *man* [ten thousand] should be included.

The objectives of grade 2 are to broaden the range of numbers, understanding how to read numbers up to 4 digits, and representing numbers and their size and order. The objectives of this grade are also to deepen an understanding that numbers can represent numbers of items and order, and to enrich the sense of numbers such as viewing numbers in relationship to other numbers, and seeing numbers as products of other numbers.

Furthermore, students are expected to understand fractions in simple cases through such activities as folding paper.

a. Counting by grouping and sorting

Since grade 1, students have been counting numbers of things by groups of two, five, or ten, or by paying attention to characteristics such as color, shape, position or type and sorting according to these characteristics, and by counting after sorting.
The activity of making groups of a suitable size, such as groups of 2, 5, or 10, leads to an understanding of the idea of multiplication. When counting numbers of many objects, the idea of making groups of ten, then making groups of ten of ten, and so on, leads to the idea of the base-10 numeration system.

b. Base-10 numeration system

In grade 1, students were taught numbers through around 120. In grade 2, the objectives are to broaden the range of numbers up to 4-digits and to let students know how to represent them in the base-10 numeration system. Also, students are taught to understand the size and order of numbers to deepen their understanding of numbers. In order to do this, it is necessary to first consider broadening knowledge to 3-digit numbers and then to 4-digit numbers.

The base-10 numeration system is the system where, when the number of a certain place becomes ten, it is traded with one of the next place, and the remaining part stays and becomes the digit of the original place. In this system, the same symbols are used in all places, and the size of the place is decided by the position of those symbols. These are the characteristics of the system.

The characteristics of this numeration system can be understood when comparing it with the act of reading numbers. For example, when reading 8235, all the place values are stated. When reading 8000, we only say the necessary place value, but need to write 0 for each place even if it is not read.

Here, in relation with comparing the size of numbers, inequality signs ">" and "<" are introduced so that the relative magnitude of numbers can be represented easily. The term "inequality sign" is introduced in grade 3.

Note (1) of "Remarks Concerning Content" states: "*man* [ten thousand] should be included." This means, for example, to teach 10,000 as the number after 9999. It is important to take this idea into consideration so the study of numbers beyond 10,000 in grade 3 will be a natural continuation and extension.

c. Relative size of numbers

"To understand the relative size of numbers" means to grasp a numbers' size in terms of units such as tens and hundreds. For example, it means to see 6000 as "600 groups of a group of 10" or "60 groups of a group of 100." The objective here is to deepen the understanding of the structure of numbers and to enrich the feel for numbers by grasping the relative size of numbers. It is important to help students understand the relative size of numbers through activities with concrete objects, and mere formal teaching should be avoided.

d. To see one number as a product of other numbers

One of the objectives is to help students understand the multiplicative structure of numbers through activities such as making equal groups when counting objects. For example, students are taught to grasp the entire size of a certain object by using a partial group of that object as a unit and then counting how many units (groups) there are.

For example, if you do an activity such as: "Arrange marbles so that it is easy to see that there are 12 marbles," various ways of arranging them can be thought of. If marbles are arranged as shown in the diagram below, they can be represented by such mathematical expressions as 2×6 , 3×4 , 4×3 , or 6×2 . This helps students see one number as a product of other numbers, deepens their understanding of numbers, and enriches their feel for numbers.



e. Simple Fractions

You can represent the number of objects using numbers like 1, 2, 3, 4, ..., but you cannot represent the size of an object cut into halves. Using fractions enables you to represent the size of an object cut into halves.

When concrete objects such as a piece of origami or a rope is cut into half, you end up with half the size of the original. $\frac{1}{2}$ is read as "one half." When you divide an object equally into two halves, one of them is the size of "one half."

If you cut a concrete object in half, then cut the resulting two objects into halves, you end up with something $\frac{1}{4}$ the size of the original object. When you divide an object equally into four parts, one of them is the size of "one fourth." If you continue further, you get $\frac{1}{9}$ the original size.

In this way, teach students to make sizes of $\frac{1}{2}$ or $\frac{1}{4}$ using concrete objects, and teach them that numbers such as $\frac{1}{2}$ or $\frac{1}{4}$ are called fractions. The meaning of fractions and their representation will be taught formally starting in grade 3. The objective in grade 2 is for students to do activities that will become the foundation of the study of fractions, and to understand the meaning of fractions empirically.

Mathematical Activities (1) a Identifying situations in everyday life where whole numbers are used.

The objective of this activity is for students to deepen their understanding of whole numbers by finding them in everyday life and to discover for themselves the value of studying whole numbers.

Students can find many examples of whole numbers in daily life. For example, numbers are used to represent amounts and quantities in things like calendars, prices of commodities, or time. Also, numbers are used to represent the result of grouping and sorting objects, such as car license numbers and room numbers.

In this way, it is important for students to discover and observe situations where whole numbers are used in everyday life, to introduce these situations to other students, and understand that whole numbers are widely used in daily life.

A (2) Addition and Subtraction

- (2) Students will deepen their understanding of addition and subtraction, and further develop their ability to use these operations.
 - a. To understand that the addition and subtraction of 2-digit numbers are based on the basic addition and subtraction of single digit numbers by exploring the ways to calculate the sums of 2-digit numbers and the differences, which are their inverse, and to be able to add and subtract accurately and reliably; to understand how to add and subtract using algorithms.
 - b. To explore ways to add and subtract simple 3-digit numbers.
 - c. To investigate properties of addition and subtraction and to utilize these properties as explore the ways to calculate and check their answers.

Remarks Concerning Content

- (2) As for content A-(1), and D-(1), () and \Box may be used when necessary.
- (3) As for content A-(2)-c, commutative and associative properties should be taught.

Building on the instructional content of grade 1, the objectives of grade 2 are to deepen students' understanding of situations where addition and subtraction of 2-digit numbers are used; for students to find calculation methods for addition and subtraction by themselves; and for students to learn and be able to use computational algorithms.

Other objectives are for students to consider how to calculate addition and subtraction of 3-digit numbers in simple cases and to be able to explain the process.

a. Addition of 2-digit numbers and subtraction as the inverse calculation

In grade 2, the first thing taught is addition of 2-digit numbers and the inverse calculation. One should help students discover how to perform calculations for themselves, based on basic addition of single-digit numbers and its inverse operation and simple cases of addition and subtraction of 2-digit numbers taught in grade 1.

For example, with a calculation such as 28 + 57, students should first understand the meaning of the calculation based on a concrete situation. Next, utilizing the calculation methods learned so far, students may obtain the sum of the ones digits, 8 + 7 = 15, and the sum of the tens digits 20 + 50 = 70 by thinking in terms of 10 as a unit. They can then reach the final answer of 85 by adding those sums. If this way of thinking is directly reflected in a paper-and-pencil algorithm—the algorithm shown on the left, an expanded version of the commonly-used algorithm—then when students learn what it means to re-group, they will be able to understand the ordinary algorithm.

^{*} [Translators' note: i.e. subtraction, including cases of (3-digit) – (2-digit) = (2-digit).]

As shown, the algorithm allows us to carry out the calculation mechanically by aligning the place values. This way of calculation is based on the base-10 numeration system and will become the basis for multiplication and division introduced later.

The same is true with teaching the subtraction algorithm. Ask students to first think about the meaning of the calculation as well as how to carry it out based on concrete situations.

When thinking about how to calculate, or checking the result of a calculation, it is important to estimate the approximate size and the number of digits in the result. This is also important in teaching calculation in later grades.

b. Simple cases of addition and subtraction of 3-digit numbers

Encourage students to think about how to calculate addition and subtraction of 3digit numbers in simple cases. Base this on the addition of 2-digit numbers and subtraction as its inverse, and on the study of numbers so far,

Examples of simple cases are as follows:

(1) Addition and subtraction of numbers using hundred as a unit

Here, addition and subtraction that uses a hundred as a unit means, for example, calculations such as 800+700 or 500–100. These calculations can be thought of as 8+7 or 5-1, by looking at numbers using 100 as a unit.

(2) Addition and subtraction with one 3-digit number and one 2-digit number, etc.

Here, addition should not involve regrouping into the hundreds place: for example, 628+7 and 234+57. Subtraction should not require regrouping from the hundreds place; for example, 753-6, 683-51, or 546-27.

By teaching simple 3-digit addition and subtraction, make sure that students understand not only calculations of 2-digit numbers, but also numbers through 3 digits. Also, in grade 3, instruction should expand this understanding of how to calculate addition and subtraction to 3- and 4-digit numbers.

c. Properties of addition and subtraction

We often group numbers or change the order of calculation. For example, when calculating 25+19+1, we think 25+(19+1) to make this calculation more efficient. Also, we can check the results of calculations by comparing the results of 16+8 and 8+16. Help students pay attention to the properties of calculation through these activities.

Note (3) of "Remarks Concerning Content" says: "As for A-(2)-c, commutative and associative properties should be taught." Therefore, teach students the commutative and

associative properties of addition. You can also have students think about the mutual relationship between addition and subtraction, and teach them to use subtraction for checking results of addition, or addition for checking results of subtraction. Checking of calculations should include both the results and the processes. This emphasizes the fact that properties of calculations should be utilized, not just memorized. A general study of properties of calculation is done in grade 4.

Note (2) of "Remarks Concerning Content" says, "() and \Box may be used when necessary." This is to encourage the ingenuity of students to deepen their understanding of addition and subtraction.

A (3) Multiplication

- (3) Students will understand the meaning of multiplication and be able to use it appropriately.
 - a. To become aware of situations in which multiplication may be applied.
 - b. To investigate simple properties of multiplication and to utilize those properties as they study basic multiplication facts and check the answers of calculations.
 - c. To learn the multiplication table and to be able to correctly multiply1-digit numbers.
 - d. To explore the ways to calculate the simple multiplication of a 2-digit number and a 1-digit number.

Mathematical Activities (1)

b. Identify patterns and properties of multiplication through the development and study of the multiplication table.

Terms/Symbols

×

Remarks Concerning Content

(4) As for the content A (3)b, the pattern of an increase in product when the multiplier increases by one and the commutative property of multiplication should be taught.

One of the objectives of grade 2 is to understand where multiplication is used and, based on this, to understand the meaning of multiplication. Also, other objectives of this grade are to compose multiplication tables based on this understanding, paying attention to properties of the multiplication table in the process; to master the multiplication table, i.e. to be able to do multiplication of 1-digit numbers with certainty; and to utilize these skills in everyday life and study.

In addition, other goals of this grade include being able to think about how to do simple cases of multiplication of a 2-digit and a 1-digit number and to explain these examples based on simple properties that hold for multiplication.

a. Cases where multiplication is used, and its meaning

Multiplication is used to find how many units there are when one unit is known. That is, multiplication is used to simplify the expression of adding the same number over and over; in other words, a concise expression of accumulation. Furthermore, the meaning of multiplication as repeated addition may be considered as a way to find the amount that is so many times as many as the base amount.

The multiplication table is not only simple in its representation, it is also easy to get results once students memorize the traditional chant that has been used in our country since ancient times.

The symbol "×" is used to represent multiplication.

b. Properties of multiplication

Note (4) of "Remarks Concerning Content" says, "As for the content A-(3)-b, the pattern of an increase in product when the multiplier increases by one and the commutative property of multiplication should be taught." Have students study, for themselves, the commutative property of multiplication, and the property that the product increases by the size of the multiplicand when the multiplier increments one at a time.

When composing the multiplication table, it is important for students to see that the product increases by the size of the multiplicand when the multiplier increases by 1, to see the commutative property of multiplication, and to use these facts effectively to compose multiplication tables or to check results of calculations.

c. Multiplication table

When teaching the multiplication table, it is important to have students comprehend it by composing one for themselves and identifying patterns in the table.

The multiplication table is the basis for the multiplication and division taught in the later grades. Therefore, it is important to make a connection between the multiplication table and experiential activities and daily experiences when having students compose and understand it. It is also important to help students master every row of the table so they can calculate correctly and use them in their everyday life and study. Students become proficient in using the multiplication table by utilizing it in their daily life and study.

d. Simple cases of multiplication of a 2-digit and 1-digit number

As simple cases of multiplication of a 2-digit number and a 1-digit number, teach multiplication of 2-digit numbers up to about 12. You can explain how to do such calculations based on the multiplication table and by utilizing the property of

multiplication where the product increases by the size of the multiplicand when the multiplier increases by 1.

For example, take a case of multiplication where 4 is multiplied by a 2-digit number. First, from $4 \times 9=36$, which is in the fourth row of the multiplication table, students see that $4 \times 10=40$ (4 greater than 36). Further, $4 \times 11=44$ (4 greater than 40) and $4 \times 12=48$ (4 greater than 44). Students can see how to obtain products this way.

Also, one can see 10×4 means four 10's, hence 40. Also one can see $11 \times 4=44$ and $12 \times 4=48$, based on the properties which hold for multiplication or by drawing diagrams for explanation.

Thinking about how to carry out simple cases of multiplication of a 2-digit and a 1digit number provides the foundation for the study of multiplication of 2- and 3-digit numbers taught in grade 3.

Mathematical Activities (1) b

Identifying patterns and properties of multiplication through the development and study of the multiplication table

The objective of this activity is to understand the properties and patterns of computation through composing and examining the multiplication table.

For example, students may find, "when the multiplier increases by one, the product increases by three" when learning the three's facts. Help students investigate if something like this happens in other cases by developing the multiplication table or examining the completed multiplication table. Students can conclude inductively that "when the multiplier increases by one, the product increases by the size of the multiplicand." Also, while composing the multiplication table, students may find that the answers of "3×4" and "4×3" are both 12. From this and similar cases, students may inductively discover this property of calculation: "Even if the multiplier and the multiplicand are switched, the answer remains the same."

Students may find various patterns in the multiplication table, such as the sum of the 3^{rd} row and the 4^{th} row is the 7^{th} row, or multiplication of the same numbers such as 1×1 , 2×2 , 3×3 , ... are aligned on the diagonal line. Teaching students to find various patterns in the multiplication table through composing and examining it will also teach them the joy of discovery.

B. Quantities and Measurement

B (1) Units of length and their measurement

- (1) Students will understand the meaning and units of linear measurement, and they will be able to measure length.
 - a. To become aware of units of length—millimeter (*mm*), centimeter (*cm*), and meter (*m*).

Mathematical Activities (1)

c. Estimating the length and volume of objects in surroundings, as well as measuring objects using standard units.

Terms/Symbols unit

A *unit* is a size that will be the basis of measurement. *Measurement* means to quantify the size of an object based on a fixed quantity.

In grade 1, students learned the basic idea of measurement: to compare the size of objects in concrete situations, to decide the appropriate length for the unit, and to represent the size of the object by counting the number of units.

In grade 2, based on what was taught in grade 1, help students be aware that it is necessary to use standard units for length, not any arbitrary length around them (arbitrary unit). Also, help them understand the meaning of units and gain the ability to measure using those units.

A ruler is used to measure length. Students are expected to understand how tick marks are organized on the ruler.

a. Units of length (mm, cm, m)

For units of length, millimeter (mm), centimeter (cm), and meter (m) will be taught here. Help students realize they can choose an appropriate unit based on the size of the object they are measuring. Also, help students pay attention to the necessity of the unit with such prefixes as milli- (m) or centi- (c) to deal with shorter lengths, and help them understand clearly the meaning and the role of units.

Mathematical Activities (1) c

Estimating the length and volume of objects in their surroundings, as well as measuring objects using standard units

The objective of this activity is for students to experience and understand the meaning of length and the idea of measurement through the actual measuring of objects.

For example, when measuring objects in their surroundings, students can find objects about 1 m long, and actually measure the length of those objects using a meter stick.

By providing experiences with these types of measuring activities, help students understand the meaning of quantities, how to measure them, and how to comprehend the standard units of length.

B (2) Units of volume and their measurement

(2) Students will understand the meaning and units of volume, and they will be able to measure volume.

a. To become aware of units of volume – milliliter (*ml*), deciliter (*dl*), and liter (*l*).

Mathematical Activities (1)

c. Estimating the length and volume of objects in their surroundings, as well as measuring objects using standard units.

In grade 2, through the activities of measuring the volume of objects, help students realize the necessity of using standard units, not arbitrary quantities in their surroundings (arbitrary units); and help them understand the meaning of units and how to use them correctly while measuring.

In grade 1, students measured volume using an object of appropriate size in their surroundings. In grade 2, they will be taught how to use things like a 1 liter measuring cup.

a. Units of volume (*ml*, *dl*, *l*)

Milliliter (ml), deciliter (dl), and *liter (l)* are introduced as units of volume. Teach 1 deciliter (*dl*) as a unit that is made by equally dividing 1 liter (*l*) into ten portions and taking one of them. Teach milliliter (*ml*) as a unit for representing volume that cannot be measured using deciliter (*dl*), and teach that one *l* is equal to 1000 *ml*.

Mathematical Activities (1) c

Estimating the length and volume of objects in their surroundings, as well as measuring objects using standard units

The objective of this activity is for students to experience and understand the meaning of volume and the idea of measurement through actually measuring objects.

For example, when measuring the amount of water that a glass bottle and a plastic bottle can hold, first let students predict how much water each can hold, then actually measure the amount of water in each bottle.

By providing opportunities to actually experience these kinds of measuring activities, help students understand the meaning of volume and how to measure it, and the sizes of standard units of volume.

B (3) Units of Time

- (3) Students will understand elapsed time and be able to use it appropriately.
 - a. To become aware of days, hours, and minutes, and to understand the relationships among them.

Elapsed time is a quantity that represents the interval from one point in time to another. Though sometimes the difference between time and elapsed time is not clear in everyday life, these two are distinguished when teaching.

a. Units of time (days, hours, and minutes)

In grade 2, *day*, *hour*, and *minute* are introduced as units of time. Teach students to use these units appropriately in concrete situations. Also help students understand the relationships among them that a day consists of 24 hours and an hour consists of 60 minutes, and how to use them.

C. Geometric Figures

C (1) Figures such as triangles and quadrilaterals

- (1) Students will be able to understand geometric figures by paying attention to the elements that compose the geometric figures through observing and composing shapes with objects.
 - a. To recognize triangles and quadrilaterals.
 - b. To recognize squares, rectangles, and right triangles.
 - c. To recognize objects that have the shape of a box.

Mathematical Activities (1)

d. Drawing and making squares, rectangles, and right triangles, as well as using those figures in tiling.

Terms/Symbols

straight line, right angle, vertex, side, face

In grade 2, students are expected know figures such as triangles and quadrilaterals, with a focus on the components of these figures. Triangles, quadrilaterals, squares, rectangles, and right triangles are taught as plane figures. Also, objects in students' surroundings that have a box shape are used to do activities that will be the foundation for understanding the solid shapes.

a. Triangles and Quadrilaterals

In grade 1, students understand plane figures intuitively and label them with informal names.

In grade 2, students are taught the rules that shapes surrounded by three straight lines are called triangles and shapes surrounded by four straight lines are called quadrilaterals. These rules classify various figures into triangles and quadrilaterals based on the number of their constituent components, i.e., sides. Students are also encouraged to pay attention to vertices of triangles and quadrilaterals.

It is important to engage students in activities involving composing triangles and quadrilaterals of various shapes and sizes using concrete objects.

b. Squares, Rectangles, Right triangles

In grade 2, teach the meaning and properties of squares and rectangles. Also, help students understand the meaning of a right angle through studying characteristics of a square and a triangle, finding shapes with right angles from their surroundings, or composing a right angle by folding paper.

Quadrilaterals that have four equal-length sides and four right angles are called squares. There are squares in many sizes, but their shapes are the same. Quadrilaterals that have four right angles are called rectangles. There are various rectangles with different combinations of length and width.

A right triangle is a triangle that can be made, for example, by dividing a square or a rectangle into two, as shown in the diagram at right.



c. Box shapes

In grade 2, teach about objects in students' surroundings that are box shaped. Also create activities that will provide the foundation for understanding solid shapes in grade 4, such as a cube and a rectangular prism.

Help students pay attention to the elements such as vertices, sides, and faces through observing concrete objects. If one observes a box-shaped object, one may notice there are faces with the shape of a square and faces with the shape of a rectangle. There are edges where two faces meet, and a vertex is a point where those edges meet.

Also, help students investigate the number of vertices, sides, and faces. To do this, it is recommended to introduce activities of observing box-shaped objects, and composing and decomposing them. For example, compose a box by gluing six rectangles or squares together, or composing a box shape using 12 sticks. Also, gathering paper boxes, cutting them open, and composing boxes again from the cut outs would be a good activity. This will lead students to realize that solid figures are composed of plane figures.

Mathematical Activities (1) d

Drawing and making squares, rectangles, and right triangles, as well as using those figures in tiling.

The objective of this activity is for students to pay attention to the composition of figures and experience and understand the characteristics of squares, rectangles, and

right triangles through drawing them on dot paper like the grid at right, or by folding paper. For example, the following activities can be suggested:

- Help students understand the elements composing figures by drawing squares, rectangles, and right triangles by connecting the points arranged on the grid like the one shown at right.
- By having students explain how to make a square by folding a rectangular piece of paper like the one at right, foster students' ability to develop an explanation that is based on the constituent components.

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- As they fold the four corners of a piece of paper like the one at right to make a rectangle, help students to pay attention to the elements which make up this shape.
- Compose a right triangle by dividing a square or a rectangle on the diagonal, or compose a large rectangle from right triangles of the same size.

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It is also important for students to identify triangles or quadrilaterals in concrete objects in their surroundings.

Furthermore, it is important to feel the broadening of a plane and the beauty of patterns generated by tiling squares, rectangles, or right triangles according to certain rules.

For example, rectangles or right angle triangles can be tiled as in the diagrams below:



Tilings of right triangles





D. Quantitative Relations

D (1) Mutual Relationship between Addition and Subtraction

(1) Students will be able to understand the mutual relationship between addition and subtraction.

Mathematical Activities (1)

e. Representing and explaining the relationship between addition and subtraction.

Remarks Concerning Content

(2) () and □ may be used when necessary according to the content in A-(2)-c and D-(1).

Suppose there are three quantities A, B, and C. If their relationship is something like the one in the diagram below, for example, then addition is the operation used to find C when A and B are known, i.e. A+B=C or B+A=C.



Also, if C and either A or B are known and you want to find B or A, then subtraction is appropriate and can be represented as C-A=B or C-B=A. Addition and subtraction are mutually related with respect to which of the three quantities is to be found. This relationship between addition and subtraction is called the mutual relationship between addition and subtraction.

Teach students the mutual relationship between addition and subtraction using situations such as (1), (2), and (3) below. For each of these situations, have students come up in front of the classroom and act out the situation, arrange concrete objects, express the situation using diagrams and mathematical expressions, make a connection between the representation with concrete objects and the representation with diagrams or mathematical expressions, so that students will deepen their understanding.

(1) Subtraction situations where addition is needed to find the missing quantity.

For example: "We had several apples. We ate 5 of them. Now we have 7 apples. How many apples did we have in the beginning?" This situation can be expressed as below, using a diagram, and \Box can be found by 5+7.



(2) Addition situations where subtraction is needed to find the missing quantity.

For example, take a situation like the following: "We had several apples. We were given 5 more apples. Now we have 12 apples. How many apples did we have in the beginning?"

(3) Missing subtrahend situation where subtraction is needed to find the missing quantity.

For example, take an example like the following: "We had 12 apples. We ate some of them. Now we have 7 apples. How many apples did we eat?"

To solve these types of problems, it is important that students pay attention to the relationship between addition and subtraction, make use of it to understand the problem, decide what kind of calculation to use, and check the answers. It is also important that students can explain their solutions using mathematical expressions.

When explaining solutions using mathematical expressions, it is important that students can connect the numbers and the operations with the contexts—for example, with the problem in (1) above: "There were several apples and somebody ate 5 of them. Now there are 7 apples left"—returning (adding) 5 apples to the remaining 7 apples can be expressed by the mathematical expression 7+5. Also, it is important to link the situation in (1) with the tape diagram mentioned above as part of the explanation. Through making these connections, students understand the mutual relationship between addition and subtraction, and grow in their ability to use mathematical expressions.

Moreover, it is necessary for students to deepen their understanding that mathematical expressions represent relationships among things and quantities concisely through this type of study.

Mathematical Activities (1) e Representing and explaining the relationship between addition and subtraction

The objective of this activity is to deepen the understanding of the mutual relationship between addition and subtraction. To do this, situations such as (1), (2), and (3) described above will be used —especially (1) and (2), which represent what is called "reverse thinking." Students will be challenged to think about how to solve these problems, represent these situations using diagrams and mathematical expressions, and explain them.

The following problem uses reverse thinking and is related to the mutual relationship between addition and subtraction. For example, in a problem such as "We

had several apples. Someone gave us 5 more. Now we have 12 apples. How many apples did we have in the beginning?" it is important to use a diagram like the one at right or to relate



various representations. Furthermore, in situation (2), for example, one would think it is necessary to return (subtract) 5 apples from the current total of 12 apples; therefore, the mathematical expression 12–5 is appropriate. In this manner, it is important to connect each part of the mathematical expression "12", "–", and "5" to the situation, or to relate these parts to a diagram and be able to explain by relating each part of the diagram with a corresponding part of the mathematical expression. It is important to foster students' ability to think and their ability to explain how to solve problems through these kinds of activities.

When using diagrams, it is important for students to abstract the quantities in the situation by representing concrete objects with a diagram, foster the ability to experience and understand the diagram, and use diagrams as "tools of thinking" and "tools of

explanation" in mathematical activities. Also, help students use diagrams in relationship to mathematical expressions or expressions with words, so they are able to think, interpret and explain them.

This type of teaching should also be done when teaching segment diagrams or number lines in later grades. It is important to deepen students' understanding of diagrams and mathematical expressions and their ability to represent, interpret, and explain them.

D (2) Multiplication expressions

(2) Students will be able to represent cases in which multiplication may be applied by using mathematical expressions and will be able to interpret those expressions.

Terms/Symbols

×

It is necessary to emphasize activities in which students represent concrete situations with mathematical expressions using the multiplication symbol, "×," interpret the expressions in view of these concrete situations, or interpret the situation and represent it using diagrams or concrete objects. Activities such as developing mathematical expressions or problems from multiplication expressions are important for deepening students' understanding of multiplication and their ability to use those expressions.

When teaching mathematical expressions, help students deepen their understanding of the meaning and appreciation for the simplicity and clarity of symbolic expressions by relating those expressions with phrases such as "one bag contains 5 oranges and there are 4 of them," diagrams with \circ or tapes, or representations with concrete objects.

When teaching how to interpret mathematical expressions, one can have students construct problems from a mathematical expression like 3×4 , such as: "A pack contains three containers of pudding. There are four packs. How many containers of pudding are there in total?" To further foster the ability to interpret mathematical expressions, it is important to make the connection between mathematical expressions and concrete situations like this, and to represent the interpretation using \circ 's, diagrams, or concrete objects.

D (3) Simple Tables and Graphs

(3) Students will be able to organize numbers and quantities from their everyday lives, represent them by using simple tables and graphs, and interpret such representations.

Students will be able to organize numbers and quantities from their everyday lives, and express them by using simple tables and graphs. Here, simple tables mean those with one viewpoint, like the one below.

Number of Flowers Blooming								
gourd	pumpkin	cucumber	eggplant					
4	3	5	2					

Simple graphs mean those with something like os to express quantity.

Students will be able to interpret characteristics from tables or graphs, such as which one has the most number of items.



In doing so, it is more important to be able to interpret the characteristics than to be able to draw tables and graphs in a mechanical way. By expressing what one has sorted and counted using tables and graphs, the comparison of quantities and interpretation of differences become easier.

Mathematical Activities

- (1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
 - a. Identifying situations in everyday life where whole numbers are used.
 - b. Identifying patterns and properties of multiplication through the development and study of the multiplication table.
 - c. Estimating the length and volume of objects in their surroundings, as well as measuring objects using standard units.
 - d. Drawing and making squares, rectangles, and right triangles, as well as using those figures in tiling.
 - e. Representing and explaining the relationship between addition and subtraction.

Terms/Symbols

unit, straight line, right angle, vertex, side, face, ×, >, <

3. The Content of Grade 3

A. Numbers and Calculations

A (1) How to represent numbers

- (1) Further extend students' ability to use numbers by deepening their understanding of the written numeration system.
 - a. To become aware of the units in the period of *ten-thousands* ("<u>man</u>" in Japanese).
 - b. To become aware of numbers that are 10 and 100 times as much/many as or 1/10 of another number, and to be able to represent those numbers.
 - c. To deepen their understanding of relative size of numbers.

Terms/Symbols

inequality sign, number line

<u>Remarks Concerning Content</u> (1) As for the content A-(1), *oku* [one hundred million] should be taught.

In grade 3, teach *man* [ten thousand] as a unit, and help students deepen their understanding of how to represent numbers. When teaching large numbers such as ten *man* and a hundred *man*, consideration should be given to preparing concrete situations or devising teaching materials.

a. Unit of man

In grade 2, students learned 4-digit numbers in the base-10 numeration system and dealt with one *man* [ten thousand]. In grade 3, the representation of a whole number is extended and taught through the unit of *man*.

When teaching the unit of *man*, it is important to help students experience and understand its size. Help students grasp that the size of one *man* is ten groups of 1000, and one bigger than 9999. Also, convey the size of one "man" as 5000 plus 5000, or 100 times 100. To see one *man* through these various viewpoints helps students understand the size of this number. In teaching the size of one *man*, it is important to enrich the sense of the size of numbers by examining large numbers in students' everyday lives.

Also, regarding the representation of numbers, even with numbers bigger than one *man*, like ten *man*, one hundred *man*, and one thousand *man*, help students express these numbers as one, ten, one hundred, and one thousand using *man* as a unit.

As for numbers bigger than one *man*, since they have little experience with counting numbers bigger than one *man*, it is important to help students understand these

numbers using the basic principles of the base-10 numeration system, and using a number-line that has been marked off by units of *man* to express those numbers on it,. By doing so, teach the term *number line*.

Number (1) in Remarks Concerning Content indicates that "As for the content A-(1), *oku* [one hundred million] should be taught." This is to serve as a bridge, connecting it to the formal introduction to the unit of *oku* in grade 4.

b. 10 times, 100 times, $\frac{1}{10}$ times a number

Teach how to express a number that is 10 times, 100 times, or $\frac{1}{10}$ times a number,

and help students deepen their understanding of the notation system.

When teaching 10 times bigger or 100 times bigger than a number, it is necessary to help students pay attention to the fact that the order of the digits of the number does not change and that the value of each digit is 10 times or 100 times greater than the original corresponding digit. For example, when 234 is made 10 times bigger, the 2 in the hundreds place goes to the thousands place, the 3 in the tens place goes to the hundreds place, and the 4 in the ones place goes to the tens place. In the same way, when making a number that is $\frac{1}{10}$ as big as another number, it is necessary to help students pay attention to the fact that the order of the digits of the number does not change and that

the value each digit is $\frac{1}{10}$ of the original corresponding digits.

c. Relative size of numbers

In grade 3, broaden the range of numbers to include the unit of *man*. It is important to help students deepen their understanding of the relative size of numbers, and also enrich their sense of numbers, by using *ju* [ten], *hyaku* [hundred], *sen* [thousand], and *man* [ten thousand] as units.

The calculation of 500+700 can be seen as 5+7, if one hundred is used as a unit. Also, since 800 can be seen as 8 if the unit of one hundred is used, then 800×5 can be seen as 8×5. Utilizing this way of viewing the relative size of numbers, students will be able to understand numbers, compare the size of numbers, and do calculations. Here, inequality signs will be introduced.

A (2) Addition and subtraction

- (2) Students will be able to add and subtract accurately and reliably and will further enhance their ability to use those operations appropriately.
 - a. To explore ways to add and subtract 3- and 4-digit numbers and to understand that those calculations are based on addition and subtraction of 1- and 2-digit numbers; to understand how to add and subtract using algorithm.
 - b. To add and subtract accurately and reliably, and to use those operations in different situations appropriately.

c. To explore properties of addition and subtraction, and to use them when considering ways of adding, subtracting, or checking answers.

Mathematical Activities (1)

a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.

Terms/Symbols

inequality signs

Remarks Concerning Content

- (2) As for the contents A-(2), (3) students should be able to calculate simple calculations mentally.
- (3) As for the content A-(2)-c, commutative and associative properties should be taught.

One of the objectives in grade 3 is to think about how to carry out calculations of 3or 4-digit numbers based on fundamental calculations students have learned previously, and for students to be able to do them reliably.

a. How to calculate addition and subtraction

Based on the addition and subtraction of 2-digit numbers, and simple cases of 3-digit numbers taught in grade 2, help students think about how to calculate addition and subtraction of 3- and 4-digit numbers.

For example, when thinking about calculating 154 + 172, help students find the method of calculation for themselves by making use of their prior learning experience of 54 + 72 = 126.

In doing so, it is necessary to teach students how to add and subtract 3 and 4-digit numbers by aligning the ones place, tens place, and so forth, based on algorithms for the addition and subtraction of 2-digit numbers taught in grade 2.

Also, students will be taught to think about how to calculate addition and subtraction of 4-digit numbers based on the method they used for calculations of addition and subtraction of 3-digit numbers.

b. Mastering calculation of addition and subtraction

Students will acquire the skill of adding and subtracting 3- and 4-digit numbers reliably, and be able to apply those calculation skills as necessary in problem situations.

When students think about how to add and subtract 3- and 4-digit numbers or check the results of their calculations, it is necessary for the teacher to help students utilize estimation of the results. For example, when calculating 389+4897, the answer 8787

might be obtained if the digits are not correctly aligned. Here, if one can see that 389 is approximately 400 and 4897 is approximately 5000, the answer will be 5400, so one will notice that 8787 is a wrong answer. In situations like this, it is important for students to be able to make these kinds of estimations on their own.

Simple mental calculations indicated in A-(2) include addition of 2-digit numbers or subtraction as its inverse operation. This type of calculation is often used in daily life and is even a necessary part of the process of multiplication and division. Mental calculations are often used to make estimates in daily life. It is important to consider these kinds of activities when teaching.

c. Characteristics that hold for addition and subtraction

In grade 2, when devising how to carry out a calculation or how to check the result of a calculation, students were taught to examine the characteristics that hold for addition and subtraction. In grade 3, when students think about how to calculate addition and subtraction of 3- and 4-digit numbers, or how to check their calculations, the students investigate the characteristics of calculations and become better at utilizing those characteristics. As indicated by note (3) of "Remarks Concerning Content," commutative and associative laws of addition are taught and students are encouraged to devise the idea of 387+ (74+26) when calculating 387+74+26. However, representing these ideas in math expressions explicitly and describing and utilizing the order of operations in general are in the content of grade 4.

Mathematical Activities (1) a

Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams

The objective of this activity is to help students think about the meaning of calculations, how to carry out calculations, and how to express their thoughts and ideas. Here, the meaning and methods for adding and subtracting 3- and 4-digit numbers will be explained.

For example, when calculating 568+437, based on the addition of 2-digit numbers such as 68+37 taught in grade 2, divide the numbers into the hundreds place, tens place, and ones place, and calculate each place individually. When thinking about how to treat the carried 1, the tens place will be "6 and 3 and carried 1, all together make 10," the hundreds place will be "5 and 4 and carried 1, all together make 10" and in this way the answer is obtained. Students will be able to express this using the diagram as shown below, and will be able to express their explanation with words as well.



A (3) Multiplication

- (3) Students will extend their understanding of multiplication. They will be able to multiply accurately and reliably and use the operation in different situations appropriately.
 - a. To understand that multiplication of 2- and 3- digit numbers by 1- or 2- digit numbers is based on basic multiplication facts by exploring ways to complete the calculations; to understand how to multiply using an algorithm.
 - b. To be able to multiply accurately and reliably, and to be able to use multiplication appropriately.
 - c. To explore properties of multiplication and use them when considering ways of multiplying or checking answers.

Mathematical Activities (1)

a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions, and diagrams.

Remarks Concerning Content

- (2) As for the content A-(2), (3), students should be able to calculate simple calculations mentally.
- (4) As for the content A-(3), calculations that involve 0 in multiplier or/and multiplicand should be taught.
- (5) As for the content A-(3)-c, commutative, associative, and distributive properties should be taught.

In grade 3, calculations such as 2- or 3-digit numbers multiplied by 1- or 2-digit numbers are taught.

a. How to calculate multiplication

When teaching multiplication where a multiplier is 1-digit number, it is important to teach students how to think about the calculation on their own. For example, when thinking about how to carry out 23×4 , we can see 23 as 20 + 3 and divide the calculation into 20×4 and 3×4 . This idea is connected to the computational algorithm. It is important for students to use the base-10 numeration system and the multiplication table that they already learned as a basis for thinking about the multiplication algorithm. The same is true when teaching calculations in which a multiplier is a 3-digit number and a multiplicand is a 1-digit number.

When the multiplier is a 2-digit number, it is possible for students to think how to calculate based on what they have previously learned: multiplication by a multiple of 10, and multiplication by a 1-digit number. For example, when calculating 23×45, one can view the multiplier 45 as 40+5 and divide the calculation into 23×40 and 23×5. When the students think about how to multiply by a 2-digit number based on multiplication by multiples of ten and multiplication by a one-digit number, it is important for the students to have foresight about the result and the calculation process.

It is important for the students to master the calculation skill of multiplication learned here and become able to use this calculation in their daily life.

b. Utilizing multiplication

There are simple cases of multiplication where the multiplier or multiplicand is the number of people or of some object. In addition, multiplication can be used in such cases as, "the price of the ribbon is 85 yen per meter. When you buy 25 meters of this ribbon, how much will it cost?" and in such cases as, "A string is divided into 4 equal lengths. Each of these lengths is 9 centimeters. How long was the string to begin with?" multiplication can be used as the reverse calculation of division. It is important to teach students how to judge situations where multiplication is used and become able to use it appropriately.

Note (4) of "Remarks Concerning Content" says, "Calculations that involve 0 in the multiplier and/or multiplicand should be taught." For example, when playing darts, if one hits the zero point area 3 times, it can be represented as 0×3 . If one cannot hit the 3-point area, it can be represented as 3×0 . Students can find the answer 0×3 by thinking about concrete situations or by returning to the meaning of multiplication and think of it as 0+0+0=0. The answer 3×0 can be found from concrete situations or by remembering multiplication principles: by writing down $3\times3=9$, $3\times2=6$, $3\times1=3$ and finding the products decrease by 3, students will be able to derive their answer, $3\times0=0$. This type of multiplication with 0 can also be used in calculations such as 30×86 or 54×60 .

Note (2) of "Remarks Concerning Content" states, "Students should be able to do simple mental calculations." This is because mental calculation may be needed in the process of multiplication and division. This is also because mental calculation is used in estimating or finding results even in daily life. "Simple calculations" here means calculations up to multiplication of a 2-digit multiplicand by a 1-digit multiplier. When teaching this, consideration should be taken so that it will not be too much of a burden

for students. Also, when teaching, it is important to help students make the best use of mental calculations when applying algorithms or making estimates.

c. Properties of multiplication

In grade 2, students learned the commutative law, and how the product changes each time the multiplier increases by one. In grade 3, students learn commutative and associative laws of multiplication as described in "Remarks Concerning Content"-(5). Also, it is important for students to understand that the change of the product when the multiplier increases by one can be expanded to such cases as $a \times (b\pm l) = a \times b \pm a$. Here, if the multiplier increases or decreases one at a time, the product increases or decreases by the size of the multiplier, and students will become able to use this law. Also, by investigating and working out the distributive law such as $a \times (b\pm c) = a \times b \pm a \times c$, students can understand that this has been used as the basis for the computational algorithm. For example, when considering the calculation of 24×3, thinking of 24×3 as combination of 20×3 and 4×3 is an example of the use of the distributive law.

It is important that students are taught to appreciate the merit of using these properties, and make the best use of them when thinking about how to calculate, when explaining their calculations, and when checking their results.

To summarize these characteristics and to understand that these hold for the calculation of decimal numbers as well is one of the contents of the fifth grade.

Mathematical Activities (1) a

Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions, and diagrams

The objective of this activity is for students to think about the meaning of the calculation and how to carry it out, and to how express their ideas. Here, doing calculations using algorithms is explained.

In the calculation of 23×4, for example, the multiplicand 23 can be divided into "20 and 3" and further to "2 tens and 3 ones." When each number is multiplied by 4, you get "8 tens" and "12 ones," thus "80+12, with answer 92." Students are expected to represent this process using a diagram as below, and become able to express it with words.

A (4) Division

- (4) Students will understand the meaning of division and be able to use it appropriately.
 - a. To become aware of different situations in which division is used: to become aware of remainders.
 - b. To understand the relationships of division to multiplication subtraction.
 - c. To calculate accurately and reliably when the divisor and quotient are both 1-digit numbers.
 - d. To explore the simple cases of division with 1-digit divisors and 2-digit quotients.

Mathematical Activities (1)

a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.

Terms/Symbols

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Division is introduced in grade 3.

a. Cases where division is used, and its meaning

Generally, there are two concrete cases where division is used. One is a case of *quotative division* where one has to find how many times more one number or quantity is than the other number or quantity. The other case, called *partitive division*, is to find one part of an equally divided number or quantity. Quotative division can be thought of as division based on the idea of repeated subtraction. For example, the meanings of $12 \div 3$ are: dividing twelve candies equally by three people (partitive division), and dividing twelve candies into groups of three (quotative division).

When comparing quotative division and partitive division, quotative division is easier to calculate, and better represented as *nozoku* [removal]. Partitive division is better represented as *warizan* [division]. When introducing division, it is important to know these characteristics well. It is also important to introduce activities that involve manipulation of concrete objects such as counters, or activities that use every day objects.

In addition, in division, there are some cases that cannot be divided completely. In such cases, teach student that remainders can be generated.

b. The relationship between division and multiplication or division and subtraction

Division can be thought of as the inverse of multiplication. Therefore, as it relates to multiplication, it is important to clarify which of the two values is being sought, the one

corresponding to the multiplier or the one corresponding to the multiplicand. Partitive division is where \Box in $\Box \times 3=12$ is sought, and quotative division is where \Box in $3 \times \Box =12$ is sought. It is important for students to realize that when we divide in the real world, we can divide things in a partitive way or a quotative way; students should thereby understand that both types of division can be expressed by the same mathematical expression.

When a remainder is involved, as, for example, in a situation such as "There are 13 cards, and these cards are to be divided equally among four people," or "There are 13 cards, and each person gets four cards," 13÷4 can be thought of as seeking \Box , which is a whole number in $\Box \times 4$ or where $4 \times \Box$ is less than and nearest to 13. And solving this problem means finding the whole number \Box that satisfies these conditions and also its remainder. We can think of 13÷4 as finding the largest number of times the cards are dealt, with the remainder being smaller than the divisor. Students are expected to understand these ideas.

c. Calculating division

Teach division where the divisors and quotients are both 1-digit numbers. For example, these are calculations that can be done by using the multiplication table once to find a quotient such as 48÷6 and 13÷4. These types of calculations will be necessary for teaching division later on where quotients are 2-digit numbers and decimals, and it will also be necessary for students to acquire these skills securely.

d. Simple cases of division where the divisor is a 1-digit number and the quotient is a 2-digit number

Utilizing division where divisors and quotients are 1-digit numbers, teach simple cases of division where the divisors are 1-digit numbers and quotients are 2-digit numbers. Here, teach calculations in the following simple cases.

One case is division where the dividend is a multiple of ten (10, 20, 30, ...) and the tens place of the dividend is divisible by a divisor as in $80 \div 4$ or $90 \div 3$. As for $80 \div 4$, it is important for students to understand for themselves, based on the idea of units, that 80 is "8 tens" and when divided by 4, the answer is "2 tens."

The other case is division where the dividend is a 2-digit number, such as $69 \div 3$. The 6 in the tens place and 9 in the ones place are divisible by 3. In the case of $69 \div 3$, based on the idea of units, students can see for themselves this calculation in the same way as the multiplication of 2-digit numbers, and, divide 69 into 60 and 9, since $60 \div 3=20$ and $9 \div 3=3$, the answer is 23.

It is important to teach this way of calculating because it ensures computation skills and the understanding of the meaning of division where the divisor and quotient are both 1-digit numbers, and fosters the ability to utilize knowledge and skills already learned.

Mathematical Activities (1) a

"Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams" is described in other contents.

A (5) The meaning of decimals and their representation

(5) Students will understand the meaning and representation of decimal numbers.

- a. To use decimal numbers in expressing quantities that are less than 1; to understand the place value of tenths and be able to write decimal numbers to the tenth place.
- b. To understand the meaning of addition and subtraction of decimal numbers through the tenths place; to explore ways to add and subtract decimal numbers and be able to calculate sums and differences through the tenths place.

Mathematical Activities (1)

- a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.
- b. Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines.

Terms/Symbols

decimal point, place of 1/10 (tenth), number line

Remarks Concerning Content

(6) As for the content A-(5) and (6), decimal numbers such as 0.1 should be taught in relation to fractions such as 1/10 by using number lines.

In grade 2, the fact that length or volume measurements are represented in units such as "9 cm and 2 mm" or "3 *l* and 6 *dl*" was taught. In grade 3, based on these experiences, students will understand the use of decimal numbers to represent quantities that are fractional parts of units, and become able to use them appropriately.

Utilizing decimal numbers enables us to express quantities less than 1 liter—as, for example, 0.3 liters, or 2 liters and 5 deciliters as 2.5 liters. When teaching this idea, help students understand that the unit structure of quantity measure has a base-10 structure.

One of the objectives here is to have students understand that decimal numbers can be used in addition and subtraction just like whole numbers, to have them think about the process of calculations, and be able to do the calculations.

a. The meaning of decimal numbers and their representation

Since decimal numbers often become necessary in relation with measurement, it is possible to introduce decimal numbers to represent quantities that are fractional parts.

One of the ideas of decimal numbers is to extend the base-10 numeration system to numbers smaller than 1. For whole numbers, when the quantity of a unit reaches ten, it is expressed as the following unit. However, in the case of decimals, the number of a certain unit (1) is equally divided into ten parts, making a new unit (0.1). The size of a quantity is represented by how many of these new units. Here, the meaning of, and the term "tenth place" is introduced. Instead of the unit of "one-tenth," it is called "first decimal place."

Expressing decimal numbers on a number line on which whole numbers are also placed is important for deepening students' understanding of decimal numbers. Teach decimal numbers in relation to the number line of whole numbers. For example, teach that 3.6 is between whole numbers 3 and 4, and furthermore, that the scale is divided evenly into ten parts between 3 and 4, and that this point (.6) rests on the 6th tick mark.

"Remarks Concerning Content"-(6) is discussed in the comment of "A-(6) The meaning and representation of fractions".

b. Addition and subtraction of decimal numbers

Teach students to understand the meaning of addition and subtraction of decimal numbers, to think about how to calculate with them, and to be able to calculate with them.

The following methods can be utilized when students think about how to calculate addition and subtraction of decimal numbers.

- (1) Think about calculation for addition and subtraction of decimal numbers by relating them to the number line.
- (2) Using relative size of numbers, represent decimal numbers with whole numbers, and calculate with those whole numbers.
- (3) By aligning decimal points, the unit of each digit is aligned as well. This enables one to calculate the whole number parts and the decimal parts separately.

Since 1 is 10 of the unit $\frac{1}{10}$, calculations involving carrying and regrouping can be done in the same way as when calculating with whole numbers.

Addition and subtraction of decimal numbers should be done based on the understanding of the mechanism of decimal numbers, by aligning the decimal points, and by calculating the whole number parts and decimal parts separately, as mentioned in (3) above. It is important for students to understand that calculations with decimal numbers can be done based on the same principle and in the same way as calculations with whole numbers.

Mathematical Activities (1) a

"Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams" and "b. Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines" are discussed in the explanation of other "Remarks Concerning Content."

A (6) The meaning and representation of fractions

- (6) Students will understand the meaning and representation of fractions.
 - a. To understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1; to understand fraction notation.
 - b. To become aware that a fraction can be considered as a collection of unit fractions.
 - c. To explore addition and subtraction of fractions in simple cases and understand their meaning.

Mathematical Activities (1)

- a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.
- b. Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines.

Terms/Symbols

number line, denominator, numerator

Remarks Concerning Content

(6) As for the content A-(5) and (6), decimal numbers such as 0.1 should be taught

in relation to fractions such as $\frac{1}{10}$ by using number lines.

In grade 2, students learned simple fractions such as $\frac{1}{2}$ and $\frac{1}{4}$. Based on this

teaching, students have gained fundamental experience in understanding fractions. Based on this learning experience, students are expected to securely acquire the meaning and the representation of fractions in grade 3. Students will learn that, just like whole numbers, addition and subtraction of fractions is possible. They are expected to think about how to carry out these calculations and be able to do the calculations.

a. The meaning and the representation of fractions

Fractions are used to represent sizes of quantities that are formed by equally dividing a whole or sizes of quantities that are fractional parts of a unit.

There are various ways to interpret the meaning of a fraction, depending on the point of view. Taking $\frac{2}{3}$ as an example, the interpretations are as follows:

- (1) expressing the quantity comprising two of the three equal parts of a concrete object
- (2) expressing a measured quantity, such as $\frac{2}{3}$ liter or $\frac{2}{3}$ meter
- (3) expressing twice the unit that is obtained when 1 is partitioned into three equal parts (i.e. the unit fraction $\frac{1}{2}$)
- (4) expressing the relative size of A when B is 1, in usage such as "A is 2/3 of B"
- (5) expressing the result (quotient) of whole number division, "2÷3."

These distinctions are a matter of convenience. Often several of these ideas are used together when teaching. The terms denominator and numerator are introduced here.

In grade 3, ways of thinking, such as in (1), (2), and (3) are used. (4) and (5) are handled in grade 5.

b. How many of a unit fraction

Contrary to decimal numbers, which represent sizes such as $\frac{1}{10}$ a unit, we can

choose any fraction suitable for the unit such as $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$. This is one of the merits of fractions.

Fractions whose numerators are 1—such as $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$ —are called *unit fractions*.

Fractions can be thought of as multiples of a unit fraction. For example, $\frac{2}{3}$ is twice the

size of $\frac{1}{3}$. $\frac{2}{3}$ is a fraction smaller than 1. $\frac{4}{3}$ is a fraction four times bigger than $\frac{1}{3}$, and bigger than 1.

It is necessary to use some ingenuity in teaching, such as putting fractions on the number line so that students can grasp the size of fractions.

Note (6) of "Remarks Concerning Content" says, "...decimal numbers such as 0.1 should be taught in relation to fractions such as $\frac{1}{10}$ by using number lines." Consideration should be taken so that students can visually experience and understand

that 0.1 and $\frac{1}{10}$ are the same size by placing them on the top and below of the same number line after studying decimal numbers and fractions.



c. Addition and subtraction of fractions in simple cases

Students are expected to understand meaning of addition and subtraction of fractions of the same denominator, and to think about how to calculate them. For simple cases, addition and subtraction of proper fractions are taught. Also, addition with sums up to 1 and subtraction as the reverse of this addition are taught.

Regarding the method of calculation, if one thinks about how to carry out addition and subtraction of fractions as addition and subtraction of numbers of unit fractions, one can see this can be done in the same way as whole numbers.

For example, in a problem involving length that requires one to find the sum of $\frac{1}{5}$

meter and $\frac{2}{5}$ meter, students are expected to be able to think about how the result

becomes three times $\frac{1}{5}$ meter (three times the unit fraction).

Mathematical Activities (1) a

"Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams" is described in other contents.

Mathematical Activities (1) b

Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines

The objective of this activity is to have students experience and understand the meaning and the size of decimal numbers and fractions intuitively. When introducing decimal numbers and fractions, the instruction should help students understand their meaning and size in relation to concrete objects, such as shown below.



For example, when teaching decimal numbers, help students realize that the amount of liquid in a 1 liter measuring cup like (1) is two of ten equally divided portions, and express this using a diagram like (2), and using that diagram as a clue, express it on the

number line like (3). Through these activities, help students understand that 0.1 is a unit that is the result of dividing 1 into ten equal parts, and help them to represent how many of this unit (0.1) is used.

It is also important to include activities where one adds another decimal number on the number line and students compare the size of the two decimal numbers by their number line positions. This is important to help students understand the meaning and size of decimal numbers, and their addition and subtraction.

For fractions, as for decimals, help students see that they represent numbers as compositions of units. Such an understanding may be developed through a series of activities like the following: Represent their previous experience that half of a half of an origami paper is one of four equal parts of the paper, with a diagram such as (4). Students relate this diagram to a tape diagram such as (5) and eventually to the number line such as (6). From these experiences, students will see that other fractions may represent numbers by how many unit fractions they comprise—in this example, the unit

is $\frac{1}{4}$, which is obtained when 1 is divided into four equal parts.



After studying decimal numbers and fractions, when comparing the size of them, help students to use a number line, as in the diagram below, and put a decimal number (0.5) and a fraction $\left(\frac{2}{10}\right)$ on it so that they can realize that using number line makes it easier to compare the sizes of a decimal number and a fraction.



A (7) The abacus

- (7) Students will become aware of how numbers are represented on the abacus (*soroban* in Japanese) and use it to calculate simple addition and subtraction.
 - a. To become aware of how to represent numbers on the abacus.
 - b. To become aware of how to add and subtract using the abacus.

Mathematical Activities (1)

a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.

The abacus is a tool for calculation that has been used for a long time in Japan. It is a very useful tool for representing numbers or doing calculations. One can represent whole numbers and decimal numbers on an abacus by assigning place values, and manipulating the beads on it. Once one understands the mechanism of an abacus, one can perform addition, subtraction, multiplication, and division using it. The abacus is taught in grades 3 and 4.

a. How to represent numbers on an abacus

In grade 3, teach how to represent whole numbers and decimal numbers on abacus. Students will become able to represent whole numbers and decimal numbers according to the base-10 numeration system. Students will be able to represent whole numbers up

to the *man* [ten thousands] place and decimal numbers to the $\frac{1}{10}$ place.

b. How to do calculations using the abacus

As for calculations, teach simple cases of addition and subtraction. It is important for students to understand how to move beads on abacus to calculate. For whole numbers, teach addition and subtraction of 1- and 2-digit numbers. Students start with such calculations as "2+1" or "3–1" which can be done just by placing beads, and proceed to calculations such as "4+3" or "6–4" which involve composing and decomposing 5, and "8+9" or "15–7" which involve carrying and regrouping. Calculations involving the *man* place such as "3 *man* [thirty thousand] + 5 *man* [fifty thousand]" are taught as well. As

for decimal numbers, simple addition and subtraction up to the $\frac{1}{10}$ place such as "2.6 +

0.3" are taught.

Mathematical Activities (1) a

"Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams" is described in other contents.

B. Quantities and Measurements

B (1) Units and Measurements of length and weight

- (1) Students will further deepen their understanding of length. Moreover, they will understand the meaning and units of weight measurement and will be able to measure weights of objects.
 - a. To become aware of the linear unit *kilometer* (*km*).
 - b. To become aware of the units of weight *gram* (*g*) and *kilogram* (*kg*).

Mathematical Activities (1)

c. Investigating the relationships among the units within each attribute (length, volume, and weight).

<u>Remarks Concerning Content</u> (7) As for the content B-(1), the unit *ton* (*t*) should be included.

The objective of grade 3 is to deepen the understanding of length, to understand weight based on the study of length and volume from grade 1, to understand the standard units of weight, and to become able to measure weight using measuring instruments.

a. Units of length (km)

In grade 3, *kilometer* (*km*) is introduced as a unit of length. Since it is hard to see the actual length of 1 kilometer, it is necessary to use its relationship to amounts such as ten times 100 *m* or 100 times 10 *m* so that students can understand. Also, it is important to have students investigate places that are 1 kilometer from school, or to help students become interested in where the unit of kilometer is used, and to have them develop a willing attitude to investigate this.

b. Units of weight (g, kg) and measurements

Students have experienced whether something is heavier or lighter than something else when holding or wearing it. Also, they have seen or heard about measuring weight using scales on occasions such as grocery shopping and measuring body weight at school.

In teaching about weight, based on these experiences that students have had, help them understand, in the same way as with other quantities, weight can be measured in terms of how many of a unit of weight. Also, it is important to enrich the sense of fundamental quantities of weight through activities where students can experience holding objects that weigh 1 kilogram. When weighing objects, if those objects cannot be weighed directly, containers may be used. In this case, the relationship "(net weight) = (total weight) – (weight of the container)" is used.

Mathematical Activities (1) c

Investigating the relationships among the units within each attributes (length, volume, and weight)

The objective of this activity is to help students become aware of the relationships common to each unit based on the study of the units of length, volume and weight up to grade 3, and to deepen their understanding of units.

In this activity, students will examine the relationships common to each unit by using a table that summarizes the units of length (millimeter, centimeter, meter, and kilometer), volume (milliliter, deciliter, and liter), and weight (gram and kilogram) that they have learned up to grade 3. Through these investigations, following points can be made:

- Both units of length and units of weight have units that involve kilo-
- Both units of length and units of volume have units that involve milli-
- 1 kilometer is 1000 meters; that is, 1000 times 1 meter
- 1 kilogram is 1000 grams, that is, 1000 times 1 gram
- 1 liter is 1000 milliliters; that is, 1000 times 1 liter
- 1 meter is 1000 millimeters; that is, 1000 times 1 millimeter

In grade 6, units taught through grade 6 will be summarized in the study of "Units and structure of the metric system."

Note (7) of "Remarks Concerning Content" says that, in addition to gram and kilogram, the unit of weight *ton* (*t*) should be taught. Here, teach that the unit of ton is often used in daily life to express heavy objects and that 1 ton is 1000 kilograms.

B (2) Selecting appropriate units or measuring instruments

(2) Students will be able to estimate length and weight. Moreover, they will be able to select appropriate units and instruments to measure length and weight.

The objective of this activity is to become able to make efficient measurements and represent them appropriately in everyday life.

When one measures some quantity, one should estimate approximately how big the object is, and select the appropriate unit and measuring instrument. For example, teach that when measuring the length of a curve, such as the distance around a tree trunk, one would use a measuring tape, and when weighing an object, one would estimate the approximate weight and select the appropriate measuring instrument. When teaching, care should be taken so that students understand that it is desirable for the measure of a quantity to be expressed with numbers that are easy to use by selecting an appropriate unit.

B (3) Clock time and elapsed time

- (3) Students will understand elapsed time.
 - a. To become aware of the unit of time *second*.
 - b. To be able to determine clock time and elapsed time, which are relevant in their daily lives.

Time is a quantity that, unlike length, area, or volume, is difficult to capture from appearances. Also, it is difficult to measure directly by deciding on a standard size and measuring it by this unit. So, it is important to help students gradually understand time by recognizing elapsed time in concrete situations, and making connections with activities and experiences in everyday life.

a. Units of time: the second

The *second* is introduced as a unit of time. The relationship between minute and second, that one minute is 60 seconds, and the ability to use these units is also taught here. It is important to teach where this unit, the second, is used in real life.

b. Determining clock time and elapsed time

Students will become able to determine clock time and elapsed time in concrete situations in their daily lives where time is necessary. Their understanding will deepen through activities with concrete objects, such as actually moving clock hands. When determining the time by calculation, care should be taken so that concrete situations in daily life are provided in which telling time is needed, and unnecessary conversions of units of time should be avoided.

C. Geometric Figures

C (1) Geometric figures such as isosceles and equilateral triangles

- (1) Students will be able to understand geometric figures by paying attention to the elements that compose them through observing and composing geometric figures.
 - a. To recognize isosceles triangles and equilateral triangles.
 - b. To recognize angles.
 - c. To recognize circles and spheres; to recognize the center, the radius, and the diameters of a circle.

Mathematical Activities (1)

d. Constructing isosceles triangles and equilateral triangles using compass and ruler.

In grade 3, isosceles triangles and equilateral triangles are introduced. Students will also understand angles in relation to these triangles.

a. Isosceles triangles and equilateral triangles

Help students grasp the features of triangles by paying special attention to the lengths of the sides of triangles. A triangle that has two sides of equal length is called an isosceles triangle, and a triangle that has three sides of equal length is called an equilateral triangle. Help students understand these isosceles and equilateral triangles through activities involving construction using compasses and rulers. Also help students to verify that an isosceles triangle has two equal-sized angles and an equilateral triangle has three equal-sized angles by engaging them in activities such as examining these triangles or cutting triangles drawn on paper and folding them.

Furthermore, help students enrich their sense of geometric figures by involving them in activities such as: tiling a plane surface using congruent isosceles triangles or equilateral triangles in order to understand that these geometric figures can tessellate; observing a pattern of tiled triangles and recognizing other geometric figures in it; or realizing that a plane figure can extend, and that there is beauty in geometric figures.



b. Angles

Right angles have been taught in grade 2. In grade 3, teach that the shape formed by two sides coming from the same vertex is called an angle. Students will become able to compare the size of angles by overlapping two angles. Students will be taught that, if they cut out isosceles triangles or equilateral triangles and fold them by overlapping the sides of equal length, they can see that the overlapping two angles are the same size.

The measurement of angles and the unit used to express the size of angles are taught in grade 4.

c. Circles and spheres

In grade 1, students were taught that circles and spheres are things that have a round shape or a ball-like shape. In grade 3, students will understand circles through activities such as



observation, sorting, composing, and drawing figures, and spheres will be understood through observation.

As for circles, such as the figure above, help students understand that all points on the circle are equidistant from the center.
Radius is defined to be the line from the center to the circle. *Diameter* is a straight line that passes through the center, from one point on the circle to another point on the circle.

Furthermore, help students recognize that there are countless radii or diameters through activities such as drawing. Also, help students get used to manipulating compasses through activities such as making designs with circles, and help them see the beauty of circles. Also, help students get interested in circles through activities such as drawing big circles in the playground, or finding circle-shaped objects in their daily lives.

The compass is used in conjunction with this content, not only to draw circles, but also as a tool for measuring and moving an equal length of a line segment. Also it can be used to compare length.

Activities such as finding the center of a circle by folding a circle made of paper, or making a top, would be effective for students to recognize the properties of circle.

As for spheres, help students understand through manipulating or observing models that the section of a sphere cut by a plane is always a circle, and that the cut section is largest when a sphere is cut exactly into halves. The size of the diameter of a sphere such as a ball can be investigated with activities like placing the ball between two solid figures such as rectangular prisms.



Mathematical Activities (1) d

Constructing isosceles triangles and equilateral triangles using compass and ruler

The objective of this activity is for students to become able to utilize the methods for drawing geometric figures using ruler and compass according to the purpose and situation, through activities like drawing isosceles triangles and equilateral triangles using a compass and ruler.

This activity is also important for helping students become good at using a ruler and compass when focusing on the constituent elements of geometric figures or understanding their properties.

The following methods can be used to draw isosceles triangles with a compass and ruler:

- How to draw using a given condition (such as the base)
 When segment BC is given, draw with a compass an arc whose center is B, and draw another arc with the same radius whose center is C. Then, connect the intersection point A with points B and C.
- (2) How to draw using circles Draw a circle with a compass. Using a property of a circle that the radii are the same length no matter where they are measured, connect two points on the circle and the center.

There is yet another way of drawing isosceles triangles using grid paper.

(3) How to draw using grid paper

Choose a segment BC on a grid, and then connect points B and C with point A on the perpendicular bisector of points B and C.

Equilateral triangles can be constructed using origami paper as shown in the diagram below:



- (1) Fold origami paper into halves (see the picture below) and put a crease in the paper.
- (2) Overlap the vertex on the lower right corner on the crease.
- (3) Mark the point where the vertex and the crease overlap.
- (4) Connect the vertex of the origami paper and the point of the mark

Composing an equilateral triangle from origami paper



(1) Make a crease



(2), (3) Fold the lower right vertex to the crease and mark the point where they meet.



(4) Connect the vertices of the origami paper and the point of the mark.

D. Quantitative Relations

D (1) Mathematical expressions of division

(1) Students will be able to represent the cases in which division may be applied by using mathematical expressions and will be able to interpret such expressions.

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Terms/Symbols
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In grade 3, students will become able to understand mathematical expressions with the symbol "÷" for division. When teaching, emphasis should be put on representing quantitative relations using mathematical expressions and interpreting mathematical expressions in the same way as was done when teaching addition, subtraction, and multiplication.

When teaching representation using mathematical expressions, while deepening students' understanding of the meaning of mathematical expressions by connecting them to phrases such as "Divide 12 candies equally among three people," to diagrams that use \bigcirc or a tape, or to the manipulation of concrete objects, help students to appreciate the conciseness and clarity of expressions using the \div symbol.

In grade 3, to interpret mathematical expressions means to grasp concrete quantitative relations from them. For example, from the mathematical expression $15 \div 3$, one can imagine a problem such as "There are 15 oranges. If you divide them so that one person gets 3 oranges, how many people can get oranges?" It is important to connect mathematical expressions with concrete situations such as this.

Also, it is important to help students understand the meaning of expressions of division in relation to those of multiplication such as "the multiplier and multiplicand in multiplication correspond to the divisor in division," while linking them to words or diagrams.

D (2) Mathematical expressions that represent quantitative relations

- (2) Students will understand mathematical expressions that represent relationship of quantities and will be able to use them.
 - a. To represent quantitative relationships using mathematical expressions and to recognize relationship among mathematical expressions and diagrams.
 - b. To represent quantitative relationships in mathematical expressions by using □ and to investigate mathematical expressions by substituting numbers for □.

As for representing and interpreting with mathematical expressions, from addition in grade 1 through subtraction, multiplication, and division, students have been taught to understand the meaning of the mathematical expressions and to be aware that the mathematical expression is an excellent way to represent quantitative relationships with conciseness, clarity, and precision, and generality. The objective of grade 3 is to raise awareness of mathematical expressions and to help students master the representation and interpretation of mathematical expressions.

a. Relating mathematical expressions to diagrams

In grade 3, when teaching mathematical expressions, students are expected to be able to represent numerical quantities and quantitative relationships using mathematical expressions in relation to concrete situations. At the same time, they are expected to master skills for utilizing mathematical expressions: to be able to interpret the meaning of the situation that the mathematical expression is representing; to be able to explain their own thinking through the use of mathematical expressions; and to be able to work and think using mathematical expressions.

When teaching diagrams, help students become able to represent numerical quantities and quantitative relationships using diagrams, become able to interpret the quantitative relationships represented by diagrams, and become able to explain one's own ideas using diagrams.

Moreover, students are expected to understand that a mathematical expression and a diagram may represent the same thing.



When teaching, it is important to help students relate mathematical expressions and diagrams through activities where students interpret quantitative relationships represented in diagrams and express them in mathematical expressions, and where students interpret quantitative relationships in mathematical expressions and express them in diagrams.

Also, it is important for students to deepen their understanding of the mutual relationship between addition and subtraction, and between multiplication and division, and to become able to explain those relationships by relating mathematical expressions and diagrams.

b. Mathematical expressions using \Box

In grade 2, in relation to deepening the understanding of addition and subtraction, the use of () and \Box is taken up in "Remarks Concerning Content".

In grade 3, students are expected to be able to express mathematical relationships according to the context by using symbols such as \Box to represent unknown quantities, and to be able to investigate numbers that correspond to \Box .

There are two cases where \Box is used: to represent an unknown numerical quantity and to represent a variable. In grade 3, the focus should be on representing unknown numerical quantities. Students are expected to be able to precisely understand quantitative relationships through writing mathematical expressions using symbols such as \Box , and through making the connection between writing such expressions and representing quantitative relationships in diagrams.

When teaching, consideration should be taken to use \Box as the placeholder where at first students fill in numerals, and then gradually use \Box as a symbol that represents an unknown numerical quantity so that they will understand the role of \Box as a letter.

When examining which number should be in \Box , for example, in the mathematical expression \Box +8=17, we can think of a method to fill in \Box with 1, 2, 3, etc., in order.

Another method is to estimate a suitable number for \Box and fill in \Box with 8, and then with 9. The most efficient method would be to use the inverse calculation based on the mutual relationships among the four arithmetic operations. It is important to help students understand, through a sufficient number of these mathematical activities, not only that the symbol \Box represents 9, but also that the mathematical expression \Box +8 itself represents the numerical quantity 17.

D (3) Tables and bar graphs

- (3) Students will be able to gather, sort, and organize data, clearly represent the data in tables and graphs, and interpret tables and graphs.
 - a. To become aware of how to interpret and draw bar graphs.

Mathematical Activities (1)

e. Organizing data by dates, locations, etc., and representing data in tables.

In grade 2, students were taught to represent what they sorted and organized in tables or simple graphs with pictures and diagrams. The main objectives in grade 3 are, based on what students have learned so far, for students to be able to decide on a viewpoint according to a purpose, sort and organize data, represent the data in tables and graphs, and interpret that data.

Also, simple 2-dimensional tables will be introduced, and students will be taught to see tables from two viewpoints such as time and place. As for tables, since there are many ways to classify, sort, and represent them, is important to understand the characteristics of each way and become able to use them appropriately depending on the purpose.

Also, in relation to tables, students are expected to be able to recognize the characteristics of a bar graph. Bar graphs enable students to capture the differences in the quantity of data at a glance, through representing what students find out for themselves.

a. How to interpret and draw bar graphs

The objectives of the study of bar graphs in grade 3 are to interpret the different sizes and differences between the sizes of numerical quantities, to understand maximum and minimum values, and to interpret the relations between elements or the overall characteristics of a group, based on what students learned in grade 2.

When teaching, it is important to teach correctly the way of choosing appropriate items and ordering them, choosing a title for the graph, etc. while taking advantage of students' ingenuity to represent what they are thinking in an easy-to-understand manner.

As for choosing and interpreting the scale of the axis, it is important to use skills in measuring by using a number line or a ruler, and the main objective is the cases where

the smallest interval is 1, 10, or 100. It is also necessary to gradually use 2 or 5 to be the size of the smallest interval, depending on purposes, and to interpret them.

In doing so, it is important to become able to use appropriate intervals depending on the purpose or the size of the grid paper through such activities as comparing several graphs that use different intervals on the same grid paper, or selecting the most suitable grid paper from several different kinds.

Mathematical Activities (1) e

Organizing data by dates, locations, etc., and representing data in tables

The objective of this activity is for students to acquire the basic and fundamental knowledge of tables and skills for creating and interpreting tables. This activity will help students understand the meaning of tables and how to use them by choosing items to sort according to the date, day of the week, time and location, etc., and organize and sort data in a skillful way, according to a purpose.

When engaging in activities, it is important to take care that students foster an attitude and ability to know exactly what the task is, willingly gather data accordingly, and sort and organize the gathered data according to their own perspective. To do this, it is important to emphasize activities such as checking data to see if there are any missing or overlapping items, or verify if there are any errors in counting so that errors will not happen easily. It is necessary to emphasize the importance of the sum total column and help students utilize their knowledge and skills of tables, for example, checking to see if the numbers in the sum total column and the numbers of the data are in agreement.

When teaching, the following points should be taken into consideration:

- (1) To make the purpose clear, to consider the conditions for the collection of data, and to select the perspective according to the purpose.
- (2) To decide which items to use and how to classify them so that there is no missing or overlapping data.

In regard to tables, there are many ways of sorting and various methods of presentation, so it is important for students to be able to use tables according to their purpose.

Mathematical Activities

- For the contents of "A: Number and Operations", "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below:
 - a. Investigating and explaining the meaning and ways to calculate with whole numbers, decimal numbers, and fractions using concrete objects, words, numbers, mathematical expressions and diagrams.
 - b. Representing and comparing decimal numbers and fractions using concrete objects, diagrams, and number lines.

- c. Investigating the relationships among the units within each attribute (length, volume, and weight).
- d. Constructing isosceles triangles and equilateral triangles using compass and ruler.
- e. Organizing data by dates, locations, etc., and representing data in tables.

Terms/Symbols

equality sign, inequality sign, decimal point, 1/10's place, number line, denominator, numerator, \div

4. The Content of Grade 4

A. Numbers and Calculations

A (1) Representing Whole Numbers

(1) Students will deepen their understanding of the base-10 numeration system.a. To understand the units in the range of *oku* [hundred million] and *cho* [trillion]; to summarize the base-10 numeration system.

Remarks Concerning Content

As for the content A-(1)-c, the cases in which large numbers are represented by putting commas after every three digits should be taught.

In grade 3, the base-10 numeration system including the unit of *man* [ten thousand]. In grade 4, teach students the units of *oku* [hundred million] and *cho* [trillion] and deepen their understanding of the base-10 numeration system.

When the range of numbers is *oku* or *cho*, it is hard to imagine the size of these numbers. Therefore, when teaching these numbers it is necessary to give some concrete examples, such as the population or budget of some country. When representing numbers bigger than 1000 *man*, the new unit *oku* is used to represent 10 times 1000 *man*. In addition to that, we introduce another new unit, *cho*, to represent 10 times 1000 *oku*. In Japan's numeration system, one, ten, hundred, and thousand are used and then repeated, with the new units of *man*, *oku*, and *cho* introduced every 4th place. Thanks to this system, we can represent numbers with many digits using a small number of units.

thousandd	hundred	ten	one	thousandd	hundred	ten	one	thousandd	hundred	ten	one	thousandd	hundred	ten	one
<i>cho</i> [trillion]			<i>oku</i> [hundred million]			<i>man</i> [ten thousand]									

Whole numbers are represented using the base-10 numeration system. This means that numbers are written based on the fundamental principles shown below:

- (1) When the quantity of each unit becomes ten, it is replaced by a new unit (the idea of base-10 notation).
- (2) Instead of using different symbols for different units, use places to indicate the differences (the idea of positional notation).

The structure of this notation system enables us to represent any number in any size using 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Note A-(1)-c of "Remarks Concerning Content" A-(1)-c talks about cases where large numbers are represented with delimiters every three digits. When expressing large numbers such as the population or budget of a country, sometimes a comma is inserted every three digits. It is necessary to teach students to read numbers represented with these commas.

A (2) Approximate numbers and rounding

(2) Students will understand approximated numbers and use them appropriately.

- a. To become aware of situations where approximate numbers are appropriate.
- b. To become aware of rounding as a way to approximate numbers.
- c. To estimate the results of computations as approximate.

Mathematical Activities (1)

a. Making appropriate estimates for computation, and evaluating the appropriateness of the results of computation.

Terms/Symbols

sum, difference, product, quotient, more than or equal to, less than or equal to, less than

Remarks Concerning Content

(2) As for the content A-(2)-c, (3) and (4) students should be able to complete simple calculations mentally. Furthermore, students should be able to use mental calculations while carrying out algorithms and estimating.

The objectives of grade 4 are to understand the meaning of approximate numbers, to manipulate numbers skillfully, and to be able to use approximate numbers for appropriate purposes. It is also important to teach so that students appreciate the significance of approximate numbers: approximate numbers help us see the size of numbers at a glance, they help us make judgment and manipulation of numbers easy, and they help us work with some foresight.

Teach terms such as "greater than or equal to", "less than or equal to", and "less than," when teaching about approximate numbers and rounding. For example, "greater than or equal to 5" means "5 or greater", and "less than or equal to 4" means "4 or less". Also, "less than 10" means "9 or less".

Students are expected to be able to use terms *sum*, *difference*, *product*, and *quotient* when referring to the results of the four arithmetic operations.

a. Cases where approximate numbers are used

Approximate numbers are used in cases such as the following:

- (1) In cases where approximate numbers are used on purpose even when the exact number is known. For example, when reporting numbers of people in a baseball stadium, an approximate number is used.
- (2) In cases such as comparing populations of cities using bar graphs. The length of each bar represents the approximate population.
- (3) In cases where it is difficult to find the exact value. For example, if you want to represent the current population of our country at this moment, an approximate number will be used.

It is important for students to understand how to use approximate numbers while clarifying their purpose. When using approximate numbers in everyday life, it is important to judge how many digits to use, according to the purpose. For example, when drawing a graph, the smallest interval is determined by the size of the paper used, and this, in turn, will determine how many digits to use.

b. Rounding to the nearest whole number

Rounding is most widely used to find rounded numbers. The procedure can be described as reading the fractional part to the nearest tick mark on a scale while measuring. Technically, rounding can be performed by just looking at the next digit to the right of the place to which the number is being rounded: if it is less than or equal to four, then round down; if it is greater than or equal to five, round up. For example, when rounding 42948 to the thousands place, look at the digit "9" which is in the hundreds place, judge that it will be rounded up, and represent the number as 43000.

c. Estimating the results of the four arithmetic operations

Here, teach the estimation of the sum, difference, product, and quotient using approximate numbers. When estimating the results of computations, as pointed out in "Remarks Concerning Content"-(2), mental calculation should be utilized.

It is important to foresee the result and avoid serious errors when estimating a sum, difference, product, and quotient. In our daily life, there are many occasions where it is convenient to estimate a sum, difference, product, or quotient using approximate numbers. For example, when we go shopping, if we know approximately how much the cost will be, we can think of how to pay appropriately. Here, students are expected to become able to estimate a sum, difference, product, and quotient appropriately using rounded numbers in concrete situations.

"To estimate appropriately" means to clarify for what purpose the approximate number is to be used, to use precision appropriate to the purpose, or to judge the approximate size of the answer.

Estimating the product and quotient plays an important role when estimating the quotient of a division whose divisor is a 2-digit number.

When teaching estimation, it is important to have students judge and calculate a approximate number to an appropriate place, according to a concrete problematic situation. Consideration should be taken so that students will not repeat only formal manipulation.

Mathematical Activities (1) a

Making appropriate estimates for computation, and evaluating the appropriateness of the result of the computation

The objective of this activity is to utilize estimation in our daily life and learning environment.

For example, let's think about the problem: "A field trip fee is 198 yen per person. There are 97 people. How much will the total fee be?" If you see 198 as approximately 200, and 97 as approximately 100, you can use 200×100, and so the estimated total fee would be 20000 yen.

The problem "Decide if you can buy three items costing 225 yen, 280 yen, and 340 yen with 1000 yen" would be another example. When answering this problem, if you over-estimate these prices and use 300 yen, 300 yen, and 400 yen respectively, the estimated total is 300+300+400=1000. Since you over-estimated, the actual total price should be less than 1000, hence you can buy these three things with 1000 yen.

A (3) Division of whole numbers

- (3) Students will deepen their understanding of division of whole numbers and will be able to calculate accurately and reliably. They will extend their ability to utilize division in different situations appropriately.
 - a. To understand that division of 2- and 3-digit numbers by 1- and 2-digit numbers is based on basic division facts by exploring ways to complete the calculations; to understand how to divide using an algorithm.
 - b. To be able to divide accurately and reliably and to be able to use divisions appropriately.
 - c. To explore the relationship among dividends, divisors, quotients and reminders, and to summarize this relationship in the following expression:
 Dividend = divisor × quotient + remainder
 - d. To explore properties of division and use these properties to think about ways of dividing or checking answers.

Terms/Symbols

\triangle quotient

Remarks Concerning Content

- (2) As for the content A-(2)-c, (3), and (4) students should be able to complete simple calculations mentally. Furthermore, students should be able to use mental calculation while carrying out algorithms and estimating.
- (3) As for the content A-(3)-d, the property of division that the quotient will remain the same when both dividend and the divisor are multiplied or divided by the same number should be taught.

In grade 4, teach students the computational algorithm of division of whole numbers.

a. How to divide

For division, in grade 3, students learned how to find the quotient by using the multiplication table once, in calculations such as $12 \div 3$ or $13 \div 3$. They also learned how to handle division where the divisor is a 1-digit number and the dividend is a 2-digit number in simple cases like $80 \div 4$ and $69 \div 3$.

In grade 4, calculations where the divisor is a 1- or 2-digit number and the dividend is a 2- or 3-digit number will be introduced. When the divisor is a 1-digit number, cases such as $96 \div 8$ and $962 \div 4$ where a 1-digit number divides a 2- or 3-digit number will be taught, and the computational algorithm will be introduced as well so that students will understand the meaning of division and how to calculate it. When teaching, encourage students to think for themselves about ways to calculate and notice that calculation is based on the division they learned in grade 3. Also, help them see that when dividing, subtraction and multiplication are also used. For example, in the calculation of $96 \div 8$, we can divide 96 into 90 and 6, and see $90 \div 8$ as $9 \div 8$, regroup the remainder of 1 to 10, add 6 to this 10, and divide the resulting 16 by 8. This viewpoint is related to the computational algorithm.

Teach the division of a 2- or 3-digit number by a 2-digit divisor such as $99 \div 12$ or 567 $\div 24$. Here, it would be difficult for students to understand if you taught the procedure of the calculation mechanically. When teaching, consideration should be taken so that students fully understand each step of the calculation. It is necessary for students to be able to use knowledge of the relative size of numbers in the calculation of division. Also, estimation will be necessary to find the quotient at each step. Abilities to estimate calculations and to do simple mental computations will be utilized. There are times when an estimated quotient is too large or too small and needs to be modified. It is especially important to carefully teach what to do when modifying the estimate of quotients.

b. Using division

The skill of performing division where the divisor is a 1- or 2-digit number and the dividend is a 2- or 3-digit number should be mastered, and students should be able to use this skill whenever necessary.

For example, division is used in the following problem: "A ribbon is 96 meters. Another ribbon is 24 meters. How many times as long is the former as the latter?" Here "the base quantity" and "the quantity to be compared" are known and "how many times" is asked. Division is also used in problems such as, "The yellow ribbon is 72 meters long and four times longer than the white ribbon. How many meters long is the white ribbon?" Here the "quantity to be compared" and "how many times" are known and "the unit quantity" is asked.

Note (2) of "Remarks Concerning Content" talks about simple mental calculations. It tells that division such as 48÷2 as the reverse calculation of the multiplication of 1-digit and 2-digit numbers should be taught as simple mental calculation, and consideration should be taken so that this will not be too burdensome for students.

c. The relationship between dividend, divisor, quotient, and remainder

Division with a remainder was taught in grade 3. Therefore, for example, students were taught to express such calculations in the form " $30 \div 4=7$ with remainder 2."

In grade 4, examine the relationships among the dividend, divisor, quotient, and remainder, and summarize the relationship in the following form:

dividend = divisor × quotient + remainder

Students need to be aware that the remainder is less than the quotient. Also, students are expected to use the relationship among dividend, divisor, quotient, and remainder when checking the answers of calculations.

d. Properties of division

Note (3) of "Remarks Concerning Content" indicates that "the property of division where the quotient will remain the same when both dividend and the divisor are multiplied or divided by the same number should be taught." This can be represented by the following mathematical expressions:

if	а	÷	b	=	С,
then	$(a \times m)$	÷	$(b \times m)$	=	С
and	$(a \div m)$	÷	$(b \div m)$	=	С

It is important to help students so that they can examine this property of division for themselves. For that purpose, it is necessary to teach activities where students create mathematical expressions of division where the quotients are the same. By doing such activities, students will become aware that the division of $350 \div 50$ can be thought of as $35 \div 5$, for example, and be able to willingly examine other similar cases. This property can be utilized in various cases such as the division of decimal numbers or fractions as it relates to numbers and calculations.

Also, it can be said that the property of division is also used to estimate a quotient when using approximate numbers. This property is further utilized in grade 5 when considering the division of decimal numbers and in grade 6 when considering division of fractions. When teaching the property of division, it is desirable to teach it in situations where division is used, or when checking how to do calculations.

A (4) Solidifying the ability to use whole number computations

(4) Students will extend their ability to use whole number computations in appropriate situations by further developing their computation skills.

Remarks Concerning Content

(2) As for content A-(2)-c, (3), and (4) students should be able to complete simple calculations mentally. Furthermore, students should be able to use mental calculations while carrying out algorithms and estimating.

The four arithmetic operations of whole numbers are taught from grade 1 through grade 4. In grade 4, students are expected to solidify their abilities in computing with whole numbers and extend their abilities in using them.

The ability to compute with whole numbers includes understanding the meaning of computation and thinking about how to carry out computations. In order to judge what kind of computation is necessary in situations involving numerical quantities, it is necessary to understand the meanings of the computations. It is also important know how to compute and to exercise ingenuity in finding the appropriate computation for the situation.

The ability to utilize computation includes mastering the basic and fundamental computational skills and utilizing them in everyday living and studying. Computational skills students have acquired so far form the basis of computations that will be necessary in daily life and study. They also form the basis for continuing on to more complicated computations.

Help students master the basic and fundamental computational skills and utilize the meaning and methods of computations they have acquired up to this point when thinking about how to perform calculations of numbers with many digits.

Also, it is an objective in this grade to connect calculations to concrete situations and utilize them in daily life and study.

A (5) The structure of decimal numbers and their calculation

- (5) Students will deepen their understanding of addition and subtraction of decimal numbers. Moreover, they will understand the meaning of multiplying and dividing decimal numbers and use the operations in appropriate situations.
 - a. To become aware of the fact that written decimal numbers are structured in the same manner as whole numbers and to deepen the understanding of relative size of numbers.
 - b. To explore ways to calculate the sum and the difference of decimal numbers and to be able to calculate accurately
 - c. To explore ways to multiply and divide decimal numbers by whole numbers and be able to multiply and divide decimal numbers by whole numbers accurately and reliably.

Remarks Concerning Content

(4) As for the content A-(5)-c, include cases in which a whole number divided by a whole number and the quotient is a decimal.

In grade 4, the objectives are to teach that decimal numbers are represented in the same manner as whole numbers and to teach addition and subtraction of decimal numbers. Also, in multiplication and division, by teaching cases where the multiplier or divisor is a whole number, students will deepen their understanding of decimal numbers.

a. The structure and relative size of decimal numbers

In grade 3, decimal numbers up to the tenths place were taught. In grade 4, teach through the use of $\frac{1}{100}$ and $\frac{1}{1000}$ as units that numbers smaller than $\frac{1}{10}$ can be represented. Also, since decimal numbers are based on the base-10 numeration system just like the whole numbers, a place to the right of another place has $\frac{1}{10}$ the value of the place to its left. A place to the left of another place has ten times the value of the place to its right. It is important to be aware of this relationship, and that comparing the size of decimals or calculating them can be done in the same way as whole numbers. The hundredths place is also called the second decimal place.

Even in cases of decimal numbers, it is also important to deepen an understanding of the relative size of numbers. The relative size of numbers means to pick a certain unit and see how many units a certain number is made up of. In the case of decimal numbers, for example, help students get used to seeing 1.68 as 168 of 0.01. To foster such a viewpoint not only deepens the understanding of decimal numbers, but helps in thinking about how to multiply and divide decimal numbers effectively.

b. Addition and subtraction of decimal numbers

In grade 3, addition and subtraction of decimal numbers up to the first decimal place was taught. In grade 4, teach addition and subtraction of a broader range of decimal numbers.

When adding and subtracting decimal numbers, it is important to understand the mechanics of decimal numbers, such as aligning the decimal points and calculating each place. It is important to teach so that students understand that the calculation of decimal numbers is based on the same principle and can be done in the same way, following the same procedures as whole numbers.

In the calculation of decimal numbers, 0.1 is ten times the unit of $\frac{1}{100}$, so we can do

the calculations that involve regrouping the same way as we do calculations of numbers to the first decimal place.

For example, when doing the algorithm of 3.7 + 2.48, if 0.01 is taken as the basic unit, one can use the same principle as calculating whole numbers. But, aligning the end of the numbers instead of place values is a common mistake. So, it is important to align place values by using the decimal point as the reference mark in the way shown below, you can calculate place by place thinking of the empty place as 0. In this way, calculating decimal numbers can be done in the same way as whole numbers.

$$\begin{array}{r}
3.7 \\
+2.48 \\
\overline{6.18}
\end{array}$$

c. Multiplication and division of decimal numbers by a whole number multiplier or divisor

Students are expected to understand the meaning of multiplication and division of decimal numbers where the multiplier or divisor is a whole number. Multiplication is used when the amount corresponding to a unit is known and the total amount for a certain number of units has to be found. That is, multiplication can be thought of as adding the same number several times. For example, 0.1×3 has the meaning 0.1 + 0.1 + 0.1. Multiplication can be used as a simple representation of repeated addition. Multiplication can also be thought of as a calculation for finding the quantity corresponding to a rate and a base quantity. The meaning of division is the inverse of multiplication, and can be explained as either finding the rate or finding the base quantity. Students are expected to learn the position of the decimal point in the product and the quotient by comparing them with cases of whole numbers. For example, since 1.2 is 12 of $0.1, 1.2 \times 4$ means 48 of 0.1. Also, 31.6 is 316 of 0.1, so $31.6 \div 4$ means 79 of 0.1.

Note (4) of "Remarks Concerning Content" says, "…include cases in which a whole number divided by a whole number and the quotient is a decimal." When a whole number is divided by a whole number, the result may or may not be a whole number. Teach students that when a whole number cannot be divided by a whole number, one can still go on dividing.

A (6) Addition and subtraction of fractions with like denominators

- (6) Students will deepen their understanding of fractions. They will also understand the meaning of addition and subtraction of fractions with like denominators; and they will be able to use addition and subtraction of fraction sin appropriate situations.
 - a. To notice that some fractions represent the same number in simple cases.
 - b. To explore ways to add and subtract fractions with like denominators; to be able to add and subtract fractions with like denominators.

Terms/Symbols

proper fraction, improper fraction, mixed number

The objectives of grade 4 are to help students deepen their understanding of the meaning and representation of fractions, understand the meaning of addition and subtraction of fractions with like denominators, and be able to calculate them.

Teach the terms and meanings of *proper fraction, improper fraction,* and *mixed number*. Also, teach addition and subtraction of improper fractions and mixed numbers beginning with proper fractions.

A proper fraction is a fraction whose numerator is less than its denominator, such as $\frac{1}{2}$ or $\frac{3}{5}$. An improper fraction is a fraction whose numerator is greater than or equal to its denominator, such as $\frac{2}{2}$ or $\frac{7}{5}$. A mixed number is a combination of a whole number and a proper fraction, such as $1\frac{2}{5}$, which is the sum of 1 and $\frac{3}{5}$. It is easier to see the size of an improper fraction if it is converted to a mixed number. For example, if $\frac{25}{3}$ is converted to $8\frac{1}{3}$, one can easily see that it is a number greater than 8. It is important to enrich students' sense of the size of fractions in this way.

Sometimes it is easier to compute with fractions if they are represented as proper fractions or as improper fractions. For example, multiplication and division of fractions taught in grade 6 will be easier to compute if they are represented as improper fractions rather than mixed numbers.

a. Fractions representing the same number

There are fractions such as $\frac{1}{2}$ and $\frac{2}{4}$ whose representations are different but are the same size. In grade 4, students are expected to notice simple cases of fractions of equal size. For example, students are expected to notice the fractions on a number line that have different representations but equal sizes.

Comparing fractions by finding a common denominator is taught in grade 5.

b. Addition and subtraction of fractions

Simple cases of addition and subtraction of fractions were taught in grade 3.

In grade 4, students are expected to think about how to add and subtract fractions with like denominators and to be able to actually perform the calculation.

For example, with $\frac{3}{5} + \frac{4}{5}$, which is an addition of proper fractions, since there are 7 $\frac{1}{5}$ s, the result can be represented as $\frac{7}{5}$.

Addition of improper fractions can be thought of in the same way. For example,

there are 13 $\frac{1}{5}$ s in $\frac{7}{5}$ + $\frac{6}{5}$, so the result can be represented as $\frac{13}{5}$ or $2\frac{3}{5}$.

Students are also expected to think about how to add and subtract mixed numbers such as $1\frac{1}{5}+2\frac{3}{5}$, and actually be able to do the calculation. One of the ways to do this calculation is to separate mixed numbers into whole number parts and fractional parts, calculate the whole number parts and fractional parts separately, and then combine the results, such as $(1+2) + (\frac{1}{5}+\frac{3}{5})$. Another way is to convert mixed numbers into improper fractions such as $\frac{6}{5}+\frac{13}{5}$ and then do the calculation.

A (7) The abacus

(7) Students will be able to add and subtract using the soroban [abacus].

In grade 3, ways of representing numbers on the *soroban* [abacus] and addition and subtraction of whole and decimal numbers are taught.

In grade 4, students are expected to deepen their understanding of the workings of the abacus. Numbers are represented based on the base-10 numeration system on the abacus. Students are expected to be able to represent the whole numbers up to the unit *oku* [hundred million] or *cho* [trillion] and up to the hundredths place for decimal numbers on the abacus.

As for calculations, students are expected to understand how to calculate by manipulating beads on an abacus. For whole numbers, students are expected to carry out addition and subtraction of 2-digit numbers. Also, they are expected to perform simple calculations involving units of *oku* or *cho* such as "2 *oku* +6 *oku*" or "10 *cho* +20 *cho*." As for decimal numbers, students are expected to perform simple addition and subtraction of decimal numbers up to the hundredths place.

B. Quantities and Measurements

B (1) Units and measurement of area

- (1) Students will understand the meaning and the units for measuring area, and they will be able to determine area by calculation.
 - a. To become aware of the units of area—square centimeter (cm^2), square meter (m^2), and square kilometer (km^2).
 - b. To investigate how to determine the area of rectangles and squares.

Mathematical Activities (1)

- b. Investigating and explaining ways to determine the area of figures composed of rectangles using concrete objects, words, numbers, mathematical expressions, and diagrams.
- c. Measuring the area of objects in their surroundings.

Remarks Concerning Content

(5) As for the content B-(1)-a, include the units are (a) and hectare (ha).

The objectives of grade 4 are to understand the meaning of units and measures, and to understand how to measure the area of squares and rectangles based on the study of area in grade 1, and to be able to actually use these units and measures. For words such as "broad" and "narrow", help students notice that these words used in phrases such as "a broad/narrow road" and do not necessarily mean the size of the area.

The quantities studied so far were measured using measuring instruments, but area is found by calculations using the length of the sides, etc. and is not measured by a measuring tool. Students need to be aware of this fact.

a. Units of area (cm², m², km²) and measurement

Students are expected to appreciate the merits of representing the size of an area using numbers in the same way as in the study of measuring liquid and volume. Teach students to quantify the size of an area by deciding on a basic unit and counting how many units are in the area. For example, when measuring area, it is convenient to use a square whose side is 1 centimeter as a unit.

Square centimeter (cm^2) , square meter (m^2) and square kilometer (km^2) are introduced as units of area. When teaching these units, consideration should be taken so that students understand that it is convenient to use the units of square meter and square kilometer when measuring large areas. For example, if you use the square centimeter as a unit for measuring the area of a classroom or a gym, the numerical value would be too large to handle with ease. Students are expected to understand that they should use units such as square meter in such cases. Also, in relation to this content, students will examine the areas around them as a mathematical activity. Furthermore, students will enrich their sense of area using data

through activities such as examining the area of their home town using km^2 .

In relation with this content, note (5) of "Remarks Concerning Content" says "include the units *are* (*a*) and *hectare* (*ha*)." The units are and hectare relate to social studies as well. Here, consideration should be taken so that students

understand that are and hectare are used when representing the area of a field because if you use the square meter as a unit, the numerical value becomes too large. When teaching these facts, the relationship between square meter, square kilometer, are and hectare should also be taught. That is, the side of the unit square of 1 *a* is ten times as

long as that of $1 m^2$, the side of the unit square of 1 ha is ten times as long as that of 1 a, and the side of the unit square of $1 km^2$ is ten times as long as that of 1 ha, and the area becomes 100 times larger respectively.

b. How to find the area of squares and rectangles

The area of the shape at right can be found by counting the number of unit squares. Here, if you move the unit square on the top left to the bottom left, the entire shape becomes rectangular. One of the objectives

here is to help students understand that the area of a rectangle can be calculated if the

lengths of its sides are known. For example, when you want to know the area of the rectangular shape at right, if you think about the meaning of the area, you can see that you can tile the entire area by unit squares and count the number of the squares. Since the unit squares are tiled regularly, you can use multiplication and easily

find the area. If you measure the length and width of this rectangle using 1 centimeter as a unit, the area is represented by (width) × (length) (or (length) × (width)) with the unit, cm^2 . Hence, you get this formula:

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Area of rectangle = width × length (or length × width)
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Also, by using this formula together with concrete diagrams, you can have students think about the change of the area when the side of a rectangle becomes twice or three times longer. It is important for students to deepen their understanding from the viewpoint of mathematical expressions, and to avoid mixing up the area and the perimeter.

The above-mentioned ideas will be the basis in grade 5 for finding the areas of a figure such as triangles and parallelograms, or volumes of cubes and rectangular prisms.





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Mathematical Activities (1) b

Investigating and explaining ways to determine the area of figures composed of rectangles using concrete objects, words, numbers, mathematical expressions, and diagrams

The objective of this activity is to help students experience and understand that if they use their understanding of geometric figures, the area of complex figures may be calculated more easily using the formulas for areas of squares and rectangles. Another objective of this activity is to foster students' disposition to make logical explanations based on what they have learned so far. "Figures composed of rectangles" means Lshape figures or concave figures.

Explaining ways to determine the areas of L-shaped figures such as the one on the lower left, for example, by using words, numbers, mathematical expressions, or diagrams means the following:



When finding the area of this shape, one explanation would be to draw grid lines such as (a) and say, "I counted the number of $1 \text{ } cm^2$ squares and found the area of the shape is $24 \text{ } cm^2$." This explanation is based on the definition of area. Another explanation would be to make the shape using construction paper and divide the shape into three parts as in (b), and decide the area of each shape by computation. Yet another explanation would be to divide the shape into two rectangles as in (c) and find the area using the formula for determining the area of a rectangle. Or, from the area of a large rectangle, you can subtract that of a small rectangle as in (d).

Mathematical Activities (1) c

Measuring the area of objects in their surroundings

The objectives of this activity are to develop the sense for the size of units, and to understand how to determine area more thoroughly by actually determining area and using units in various everyday situations. It is also the goal for students to feel and experience the value of studying area through these kinds of activities.

There are students who make an error and think $1 m^2 = 100 cm^2$ by analogy and from their experience with the conversion of the units of length, even though they know that $1 m^2 = 10000 cm^2$ through computation. This kind of error can be prevented if one has a sense of how big $100 cm^2$ actually is and really understands the meaning of $1 m^2$. So measuring the areas of squares and rectangles found in the students' environment will help. The objects can be origami paper, the surface of their desks, the floor of the classroom, flowerbeds, or the gym. When doing this activity, it is necessary to have students have a sense of purpose according to their actual level of experience and help

them appreciate that the study of area is useful in their daily life, for example, through the comparison of the areas of their classroom and the multipurpose room.

B (2) Size of Angles

- (2) Students will understand the meaning and the unit for measuring angles, and they will be able to measure angles.
 - a. To conceive the size of an angle as the amount of turn.
 - b. To become aware of the unit of angle measurement *degree* ($^{\circ}$).

In the domain of "C. Geometric Figures," right angles were taught in grade 2. In grade 3, students focused on angles that were the same size in relation to figures such as isosceles triangles.



In grade 4, teach students to see angles as rotations, and the use of the *degree* (°) as the unit of their measurements.

a. The size of a rotation

The shape formed by two sides from one vertex is called an *angle*. The amount of rotation of one side, with a vertex as the center, is the size of the angle. The size of an angle is considered to be how much the two sides are apart. Teach about the size of an angle as described above.

The size of two angles can be compared directly by overlapping those angles. It is necessary to pay attention to the fact that the size of an angle has nothing to do with the length of the sides.

b. The unit of angle measure

Degree (°) is introduced as the unit of angle measure. Teach that the right angle is 90° and that one full turn is 360° .

Students will learn how to measure an angle or how to make angles of a given size using a protractor.

In addition, students will enrich their sense of angles such as the ability to judge if the size of an angle is bigger than 90° using a right angle as a benchmark.

C. Geometric Figures

C (1) Parallelograms, rhombuses, and trapezoids

- (1) Students will deepen their understanding of geometric figures by paying attention to the elements that compose them and their positional relationships, through observing and composing geometric figures.
 - a. To understand relationships such as parallelism and perpendicularity of lines.
 - b. To recognize parallelograms, trapezoids and rhombuses.

Mathematical Activities (1)

d. Investigating properties of geometric figures by tiling the plane with parallelograms, rhombuses, and trapezoids.

Terms/Symbols

 \triangle Parallel, perpendicular, diagonal

In grade 4, teach the constituent elements of geometric figures such as parallel and perpendicular lines. Also teach about parallelograms, rhombuses and trapezoids.

a. Parallel and perpendicular lines

In grade 2, squares and rectangles were taught. Since students carried out activities where they observed and constructed squares and rectangles, they have gained experience that will form the basis for understanding parallel and perpendicular lines.

In regard to two parallel lines, one can say, "When two lines do not cross no matter how far they are extended, these two lines are parallel." But the expression "no matter how far they are extended" and its meaning may be difficult for students to understand. So, it may be better to define the perpendicular relation first and then define the parallel relation as follows.

- (1) If two lines cross at right angles, it is said these two lines are *perpendicular*.
- (2) When there are two straight lines that are perpendicular to another line, it is said those two lines are *parallel*.



It is a good idea to include activities such as finding two parallel lines or two perpendicular lines by observing concrete objects around students. Also, activities such as making two parallel or perpendicular lines using concrete objects such as sticks or drawing two parallel or perpendicular lines using two set squares would be helpful. In addition, when considering figures such as parallelograms, help students deepen their understanding of figures by examining the relationship of opposite sides or how diagonals intersect.

If we define parallel lines as we did in (2) above, the fact that "two lines do not cross no matter how far they are extended" becomes one of the properties of parallel lines. Another property of parallel lines is "the distance between parallel lines is always the same."

As for perpendicularity, it is necessary to pay attention to the fact that perpendicularity represents a positional relationship between two lines and thus differs from the right angle, which is a shape.

b. Parallelograms, rhombuses, and trapezoids

By paying attention to the positional relationships and lengths of the sides, one can see the properties of quadrilaterals in the following examples, and be able to classify quadrilaterals.

A quadrilateral with two pairs of opposite sides that are parallel is called a *parallelogram*.

A quadrilateral whose four sides are equal in length is called a *rhombus*.

A quadrilateral with one pair of opposite sides that are parallel is called a *trapezoid*. Students are expected to know these names of quadrilaterals and be able to use them. It is important to do activities such as the following to help students understand these figures: sort and organize quadrilaterals by common properties by observing various quadrilaterals students construct themselves; examine the property of each figure; and draw shapes and differentiate them based on their definitions or properties. Also, teach students to find concrete objects from their surroundings that are shaped like a parallelogram, rhombus, or trapezoid.

Teach the term and meaning of *diagonal*. A *diagonal* of a quadrilateral is a line segment that connects two opposite vertices.

Parallelograms have the property that opposite sides are the same length. They also have the property that opposite angles are the same size. Another property of parallelograms is that the two diagonal lines bisect each other. These properties can be verified by cutting a parallelogram along its diagonals into several pieces and overlapping the resulting triangles, and by other similar activities.

As for rhombuses, one of their properties is that opposite angles are the same size. Drawing a rhombus and cutting and folding it can verify this property. Also, students are expected to notice properties of rhombuses such as the fact that the two diagonals are perpendicular to each other and bisect each other. Teach students to pay attention to the fact that a rhombus is a quadrilateral that has all the properties of a parallelogram.

Mathematical Activities (1) d

Investigating properties of geometric figures by tiling the plane with parallelograms, rhombuses, and trapezoids

The objective of this activity is for students to deepen their understanding of shapes experientially by focusing on the constituent elements of shapes and their positional relationships.

The objective of this activity is to enrich the sense of and ways to observe geometric figures. Students can verify that a plane surface can be tiled with parallelograms, rhombuses, and trapezoids, and identify other geometric figures or notice the properties of parallel lines in the resulting tiling patterns.



For example, if one tiles a plane with trapezoid ABCD, the result will be the image above. This verifies that the plane can be tiled by trapezoids. One can find parallelograms in the resulting pattern. Also, one may notice that if angles A and B or angles C and D are combined, the results are 180° in both cases.

Moreover, coloring tiled shapes and adding various designs to them will teach the students the beauty of geometric figures.

C (2) Solid figures such as cubes and rectangular prisms

- (2) Students will be able to understand solid figures through observing and composing solid figures.
 - a. To recognize cubes and rectangular prisms.
 - b. To understand parallelism and perpendicularity of lines and planes in connection with a rectangular prism.

Terms/Symbols

plane

Remarks Concerning Content

(6) As for the content C-(2), drawing sketches and nets of shapes should be taught.

In grade 2, students were taught to focus on the constituent elements of solid shapes by observing or composing box shapes. The objective of grade 4 is to know about cubes and rectangular prisms, and to understand solid shapes. Also, students are expected to enrich their ways of considering geometric figures in the context of rectangular prisms through observing, composing, and decomposing shapes, and to understand the parallel and perpendicular relationships between lines and planes.

The term *plane* is shown in "Terms/Symbols." It is important for students to be able to express, explain and represent using this term.

a. Cubes and rectangular prisms

A *cube* is a solid shape that is enclosed by six squares. A *rectangular prism* is a shape that is enclosed by six rectangles.



As for cubes and rectangular prisms, teach students to clearly understand the characteristics of these shapes by considering their components: by focusing on the number of edges, faces, and vertices; by observing the shapes of their faces; and by noticing parallel and perpendicular relationships of edges and faces.

Also, note (6) of "Remarks Concerning Content" says that "drawing sketches and nets of shapes should be taught" in relation to cubes and rectangular prisms. This is because one of the objectives is to help students understand how edges or faces connect, and how their spatial relationships work, through representing cubes and rectangular prisms with sketches and nets.

Sketches and nets are one of the ways to represent solid figures on a plane. Teaching students the value of this is important. Consideration should be taken in relation to plane figures when teaching this. It is important to teach that not only one, but several nets can be drawn from one solid figure. It is also important to help students imagine the solid figure that results from a given net.

b. Parallel and perpendicular relationships of lines and planes

Students are expected to understand the parallel and perpendicular relationships among lines and planes in relation to rectangular prisms. As for the edges and faces of a rectangular prism, it is important for students to be able to understand that of 12 edges, 4 are parallel, and that there are 3 groups composed of 4 edges each, that each edge is perpendicular to two faces, and that three edges which meet at one vertex are perpendicular to one another. Students can understand this through activities such as observing, composing, and decomposing.

C (3) How to represent the position of objects

(3) To understand how to represent the position of an object in space.

As for the position of an object, representing positions using words such as "front" and "rear," "right" and "left," and "above" and "below" was taught in grade 1.

In grade 4, students are expected to understand how to represent objects on a plane or in space.

To express a position in a plane, two elements (vertical and horizontal distance) are necessary. For example, marching 3 meters forward and 4 meters sideways from one of the four corners of the gym could be represented as (forward 3 meters, sideways 4 meters).

Also, to represent an object in space, 3 elements (vertical distance, horizontal distance, and height) are necessary. If one thinks of the classroom as a rectangular prism and wants to represent an object dangling from the ceiling of the classroom, and the object is located 3 meters and 4 meters from two edges connected to a common corner of the classroom floor and its height is 2 meters from the floor, one could represent the place as (3 meters, 4 meters, 2 meters), for example.

D. Quantitative Relations

D (1) Two quantities that vary simultaneously

- (1) Students will be able to represent and investigate the relationship between two quantities as they vary simultaneously.
 - a. To represent how the quantities vary on a broken-line graph and to interpret the features of their variation.

Mathematical Activities (1)

e. Identifying quantities in their surroundings that vary simultaneously, and investigating their relationships by representing them in tables and graphs.

The objective of grade 4 is to foster the ability to pay attention to quantities that vary simultaneously in concrete settings and to make their relationship clear by representing them using tables and graphs.

a. How quantities vary, and broken-line graphs

In grade 4, students are expected to make tables, express functional relationships using broken-line graphs, and interpret characteristics in the change of two quantities that are in a functional relationship. Elaborate on the idea of function through these activities. To express the changes using a broken-line graph means to place one of the two quantities that are in a functional relationship on the horizontal axis and the other on the vertical axis, to express those two quantities using points on the coordinate plane, and to show how each part changes or how the graph as a whole changes visually by connecting those points.

Also, to interpret characteristics of change on a broken-line graph means to visually perceive the change of one of the two quantities which vary simultaneously while the other increases, and to make the relationship between two varying quantities clear. To do this, it is necessary to see if quantities increase or decrease from the gradient of line.

Mathematical Activities (1) e Identifying quantities in their surroundings that vary simultaneously, and investigating their relationships by representing them in tables and graphs

The aim of this activity is to be able to utilize tables and graphs, for example, to find two quantities that vary simultaneously from the surroundings and to represent these quantities using tables or broken-line graphs, and to investigate the relationships between these quantities.

In solving problems involving quantities or geometric figures, it is important to relate what one wants to find to other quantities or figures. For example, it is important to know the relationship between the unknown and other quantities or to know what quantity (or quantities) will determine another quantity.

Also, it is necessary to make relationships clear between two simultaneously changing quantities. To do this, it is necessary to do activities such as finding many pairs of those two quantities and sorting and organizing those pairs in a table, or representing them using graphs and investigating their relationships.

It is important to enhance students' functional thinking and their disposition to consider phenomena statistically, as well as to help them experience the usefulness of such perspectives. It is also important to foster the disposition to willingly use these perspectives in their daily life and study.

D (2) Mathematical expressions representing quantitative relations

- (2) Students will understand mathematical expressions that represent quantitative relationships and be able to use them.
 - a. To understand mathematical expressions that contain some of the four basic operations and parentheses (), and to be able to calculate them.
 - b. To understand the idea of a formula and to use formulas.
 - c. To represent numbers and quantities in mathematical expressions by using
 - \Box and \bigtriangleup , and to investigate the relationships by substituting numbers for

 \Box and \triangle .

Students have been taught to represent addition, subtraction, multiplication, and division using mathematical expressions and to interpret those expressions by grade 3.

The main contents of this grade are to understand mathematical expressions composed of a mixture of those four operations as well as expressions including parentheses, to generalize quantitative relationships using formulas, and to represent quantitative relations in mathematical expressions with symbols such as \Box and \triangle .

The objectives of grade 4 are to represent quantitative relationships in mathematical expressions, to improve in their ability to interpret mathematical expressions, to understand the order of operations, and to utilize mathematical expressions appropriately. Moreover, students should acquire ways of thinking about formulas based on already-learned mathematical expressions and on expressions representing concrete situations.

a. Expressions including some of the four operations and parentheses.

The objectives of grade 4 are not only to calculate mathematical expressions but also to be able to represent relationships between quantities using expressions including a mixture of operations and parentheses; to interpret such mathematical expressions; to appreciate the merits of mathematical expressions; and to use them appropriately.

Expressions that contain some of the four operations or parentheses were dealt with previously. The main objective is to help students understand that parentheses are sometimes used to represent one quantity, and expressions that include multiplication or division represent one quantity. Students are to solidify their understanding of these things through various problems, by representing these relationships in expressions, and by interpreting problems and general relationships from expressions.

When teaching, help students understand the rules of calculation—that multiplication and division are calculated before addition and subtraction, and expressions in parentheses should be calculated before anything else—so that they master these rules. Moreover, it is important to understand that by representing relationships of quantities using expressions that contain more than one of the four operations and parentheses, the relationships can be represented concisely. It is also important to understand the merit of this, and to be able to represent relationships between quantities in one expression using the four operations and parentheses.

b. Formulas

In grade 4, in expressing the relationships between concrete quantities by using formulas, students are expected to represent quantitative relations using mathematical expressions; to be able to interpret those expressions; and to appreciate the merit of formulas since they can be used in other situations.

Formulas taught in grade 4 are not limited to those that are generally considered as formulas. They also include expressions with words used naturally when writing equations to solve concrete problems. As for general formulas, it is important to understand that they represent quantitative relationships using words, and these words can be replaced by various numbers.

As for formulas, it is necessary to emphasize the importance of the process of their derivation: making many combinations of numbers, finding the common relationships

and rules among those numbers, and representing the relations and the rules using expressions with words.

In order to help students understand that formulas represent general relationships among quantities, it is necessary to utilize activities of replacing what is represented in formulas with various numbers in concrete situations. Moreover, it is important to represent quantitative relations using formulas and to interpret formulas in concrete situations. Consideration should be taken so that students understand the merit of formulas that generalize quantitative relationships through these activities.

In grade 4, the formula for finding the area of rectangle is introduced. For example, the idea of generalizing and making a formula—such as (area of rectangle) = (height) × (width)—is often used in mathematics and various other fields. It is important to help students understand that formulas represent general relationships that hold for all numbers. It is also important for students to understand that the formula may be used to find a height, given area and width, as well as finding an area, given height and width.

c. Expressions with \Box or \bigtriangleup

In grade 3, students learned to represent unknown numerical quantities using \Box , or to write a mathematical expression using \Box .

The objective of grade 4, based on past understanding, is to become able to appropriately use mathematical expressions including symbols such as \Box and \triangle to represent variable quantities.

In grade 4, there are cases where expressions with \Box or \triangle are handled; for example, representing the relationship between one side and the perimeter of the square using $\Box \times 4 = \triangle$. Representing variables using \Box or \triangle in an expression has the merit of simplifying how relations between quantities are expressed.

When teaching, consideration should be taken so that students understand that various numbers can be substituted in place of \Box or \triangle , and that if one of either \Box or \triangle is determined, the other quantity is also determined.

Expressions with \Box or \triangle can also be used to summarize or explain properties of the four arithmetic operations. In the case where two or more symbols such as \Box , \triangle , and \bigcirc are used in one expression, it is important to have students understand that the same number is used for the same symbol. It is also important to take care that students appreciate the merit of using symbols such as \Box , \triangle , and \bigcirc , to express the quantitative relations and the computation rules simply, clearly, appropriately, and generally.

D (3) Properties of the four arithmetic operations

(3) Students will deepen their understanding of the properties of the four basic operations.

a. To summarize commutative, associative, and distributive properties.

By grade 3, students learned that commutative, associative, and distributive properties hold in concrete situations, while studying how to add or multiply, and checking the result of those calculations.

In grade 4, within the range of numbers and calculations learned so far, students are expected to summarize the properties of the four basic operations and to be able to utilize them when necessary.

a. Commutative, associative, and distributive properties

Commutative, associative, and distributive properties are represented by the following mathematical expressions:

Commutative Properties	Distributive Properties
$\Box + \triangle = \triangle + \Box$	$\Box \times (\bigtriangleup + \bigcirc) = \Box \times \bigtriangleup + \Box \times \bigcirc$
$\Box \times \triangle = \triangle \times \Box$	$\Box \times (\bigtriangleup - \bigcirc) = \Box \times \bigtriangleup - \Box \times \bigcirc$
	$(\Box + \bigtriangleup) \times \bigcirc = \Box \times \bigcirc + \bigtriangleup \times \bigcirc$
Associative Properties	$(\Box - \Delta) \times \bigcirc = \Box \times \bigcirc - \Delta \times \bigcirc$
$\Box + (\bigtriangleup + \bigcirc) = (\Box + \bigtriangleup) + \bigcirc$	
$\Box \times (\bigtriangleup \times \bigcirc) = (\Box \times \bigtriangleup) \times \bigcirc$	

In grade 4, students are expected to deepen their understanding of the properties of the four basic arithmetic operations in calculating whole numbers by trying to simplify the calculation using the commutative, associative, and distributive properties, or by finding the distributive property in the algorithm of multiplication, and to become able to utilize these properties as necessary. Also, students are expected to check that the above properties that hold for whole numbers also hold for decimal numbers.

D (4) Collecting, sorting, and organizing data

- (4) Students will be able to gather, sort, and organize data according to their purposes, to clearly represent data in tables and graphs, and to investigate features of data.
 - a. To investigate data by sorting and organizing from two viewpoints.
 - b. To become aware of how to interpret and draw broken-line graphs.

Remarks Concerning Content

(7) As for the content D-(4), ways to check for omissions and duplications when investigating data should be taught.

The objective of grade 4 is to develop students' ability to collect, sort, and organize data depending on the data and to grasp characteristics and tendencies of the data.

a. Sorting and organizing from two viewpoints

In grade 4, students are expected to become able to sort and organize data from two viewpoints simultaneously and to represent that data using tables and graphs.

When collecting and sorting data, it is necessary to group the data, depending on the purpose, by sorting the data according to some perspective. For example, suppose one is viewing data from two different points of view: from viewpoint A, it could be said, "The data have characteristic *a*" or "The data do not have characteristic *a*," and from viewpoint B, it could be said, "The data have characteristic *b*," or "The data do not have characteristic *b*," or "The data do not have characteristic *b*." If these two perspectives are considered simultaneously, the data can be divided into four categories.

It is important to view things from two different viewpoints and to sort them or to logically investigate possible situations, and to be careful that no possible cases are missed and that no case is counted twice.

Note (7) of "Remarks Concerning Content" says, "Ways to check for omissions and duplications when investigating data should be taught." As for "ways to check for omissions and duplications," consideration should be taken to teach concrete ways to avoid omissions and duplications such as counting systematically so that no data will be omitted, organizing possible cases in advance, or marking items already counted to avoid duplicating the counting. It is necessary to foster the disposition to eliminate errors to get correct results.

b. How to draw and interpret a broken-line graph

In grade 4, as is stated in D-(1), teach that a broken-line graph is used for representing functional relationships. The objective here is to use a broken-line graph to represent data according to a purpose, and to interpret and examine that data.

When teaching about broken-line graphs, it is important to help students decide on the overall size of the graph and interval of tick marks based on the size of the paper or the purpose. Help students notice the fact that the appearance of the graph changes even though they represent the same data if they change the scaling of the vertical axis, etc.

Mathematical Activities

- (1) For the contents of "A: Number and Operations," "B: Quantities and Measurement," "C: Geometric Figures," and "D: Quantitative Relations," instruction should incorporate mathematical activities such as those listed below.
 - a. Making appropriate estimates for computation, and evaluating the appropriateness of the results of computation.
 - b. Investigating and explaining ways to determine the area of figures composed of rectangles using concrete objects, words, numbers, mathematical expressions, and diagrams.
 - d. Investigating properties of geometric figures by tiling the plane with parallelograms, rhombi, and trapezoids.

e. Identifying quantities in their surroundings that vary simultaneously, and investigating their relationships by representing them in tables and graphs.

Terms/Symbols

sum, difference, product, quotient, greater than or equal to, less than or equal to, less than, proper fraction, improper fraction, mixed number, parallel, perpendicular, diagonal line, plane

5. The Content of Grade 5

A. Numbers and Calculations

A (1) Properties of whole numbers

- (1) Students will deepen their understanding of the properties of whole numbers.
 - a. To become aware of the fact that whole numbers may be classified as even and odd numbers.
 - b. To become aware of factors and multiples.

Terms/Symbols

greatest common divisor, least common multiple

Remarks Concerning Content

As for the content A-(1)-b, the greatest common divisor and the least common multiple should be taught in concrete settings; avoid putting too much weight on finding these procedurally. Also, prime numbers should be discussed in the process of investigating divisors.

Teach even and odd numbers, and divisors and multiples, and help students deepen their understanding of the properties of whole numbers.

a. Even numbers and odd numbers

When a whole number is divided by 2, the remainder is either 0 or 1. A whole number whose remainder is 0 when divided by 2 is called an even number and a whole number whose remainder is 1 is called an odd number.

In this way, whole numbers can be divided into two sets, even numbers and odd numbers. That is, any whole number belongs either to the set of even numbers or to the set of odd numbers.

There are situations in our daily life where we can utilize even and odd numbers. For example, if you want to divide students of the class into two groups, you can use students' ID numbers. When you look at the students' ID numbers as whether they are divisible by 2 or not, each student will belong to one group or the other.

b. Factors and multiples

The objectives are to teach the meaning of *factor* and *multiple*, and to help students think of factors or multiples of a number as a set.

The set of the common factors or common multiples of two whole numbers is composed of common elements of the set of factors or the set of multiples of each whole number. For example, factors of 8 are 1, 2, 4, and 8. Factors of 12 are 1, 2, 4, 6, and 12.

From this, we see that the common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is the largest of the common factors, which is 4 in this case.

Also, multiples of 8 are 8, 16, 24, 32, and so on. Multiples of 12 are 12, 24, 36, and so on. From this, we can see that common multiples of 8 and 12 are 24, 48, 72, and so on. The least common multiple is the smallest of the common multiples, 24 in this case.

Help students deepen their understanding of the properties of whole numbers by using the idea of factors and multiples in daily life.

"Remarks Concerning Content"-(1) talks about how to treat the greatest common factor and the least common multiple. These should be taught in concrete situations so that students understand their meaning. Also, as for prime numbers, help students enrich their sense of how to see whole numbers by understanding that there are whole numbers that have only two factors, 1 and the number itself.

A (2) Notation system of whole and decimal numbers

- (2) Students deepen their understanding of the base-10 numeration system for whole numbers and decimal numbers, and to be able to use this understanding effectively in computation and for other purposes.
 - a. To determine numbers that are 10, 100, 1/10, and 1/100 times as much as the original number.

a. Numbers that are 10, 100, $\frac{1}{10}$, and $\frac{1}{100}$ times as much as the original number

The main objectives of grade 5 are to summarize the properties of the base-10 numeration system and to understand them, and to be able to use them efficiently in calculations, based on the standpoint that both whole and decimal numbers are represented using the base-10 numeration system.

When summarizing the properties of the base-10 numeration system, teach that by moving the decimal point, we can make numbers 10 times, 100 times, $\frac{1}{10}$ times or $\frac{1}{100}$ times as big as the original one. In the base-10 numeration system, if you move the decimal point one digit to the right, the resulting number is 10 times as big as the original one. If you move the decimal point 1 digit to the left, the resulting number is $\frac{1}{10}$ times or $\frac{1}{10}$ times as big as the original one. That is, we can make numbers 10 times, 100 times, $\frac{1}{10}$ times or $\frac{1}{100}$ times as big as the original one by moving the decimal point. This idea is utilized in the calculation of multiplication and division of whole numbers and decimal numbers. It is important to teach in a way that students can substantially use this idea in estimation as well. As necessary, students will also make numbers 1000 times or $\frac{1}{1000}$ times as big as the original number.

When examining the relationship among sizes of numbers, do not teach only procedural manipulation. Consideration should be taken so that students can enrich their sense of the size of numbers and of their structure.

A (3) Multiplication and division of decimal numbers

- (3) Students will deepen their understanding of multiplication and division of decimal numbers, and be able to use them appropriately.
 - a. To understand the meaning of multiplying or dividing by decimal numbers based on the calculation ideas for multiplying and dividing decimal numbers by whole numbers.
 - b. To explore way to multiply and divide by decimal numbers, and be able to calculate accurately; to understand the size of numbers.
 - c. To understand that the same properties of multiplication and division for whole numbers will apply to decimal multiplication and division.

Mathematical Activities (1)

a. Investigating and explaining the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines.

The objective of grade 5 is to broaden the meaning of multiplication and division so they can be used when the multiplier or divisor is a decimal number. It is important for students to be able to develop a new method of calculation, based on the meaning and calculation methods for multiplication and division with whole numbers.

a. The meaning of division and the multiplication of decimal numbers

The meaning of multiplication of decimal numbers

The multiplication of whole numbers is used "to determine the total amount when the unit amount is known," or "to determine the total quantity which is so many times as big as a given quantity."

In grade 5, multiplication is extended to cases where the multiplier is a decimal number. Also help students be able to use multiplication in a wider range of situations and meanings by thinking about the relationship with division, and to generalize them.

We will use the same mathematical expression to represent a relationship between two quantities whether or not those quantities are whole numbers or decimal numbers, if the context of the problem involves the same relationship.

For example, when you buy 2 meters of cloth that cost 80 yen per meter, you can represent this using a mathematical expression such as 80×2. In the same way, in the case of the following problem, "You bought 2.5 meters of cloth that cost 80 yen per meter. How much is the price?" since the length of the cloth is 2.5 times longer, the price will also be 2.5 times more, and we can represent this as 80×2.5.

As seen from the above, the meaning of the multiplication of whole and decimal numbers is represented as $B \times P = A$, where B is the base quantity, P is the relative value, and A is the corresponding quantity that has the specified relative size with respect to B.


The fact that the product becomes smaller than the multiplicand B when the multiplier P is smaller than 1, can be explained using a number line.

The meaning of division of decimal numbers

As the reverse of multiplication, one of the meanings of division is to find the relative value. The other meaning of division is to find the base quantity.

When B is the base quantity, P is the relative value, and A is the corresponding quantity with the specified relative size with respect to B, the meanings of division are to find the relative value or to find the base quantity, as the reverse of multiplication. This can be seen in the following two cases:

(1) $P = A \div B$

The meaning of this division is quotative division, where the object is to find how many times A is compared to B. For example, in the case of the following: "A red ribbon nine meters long is how many times as long as 1.8 meters of blue ribbon?" the mathematical expression for this is 9÷1.8.

(2) $B = A \div P$

This is to find the base quantity; if P is a whole number, this is "fair share" division. For example, in "Some cloth costs 200 yen for 2.5 meters. How much is 1 meter of this cloth?" the mathematical expression for this is 200÷2.5.

It is important to understand that these mathematical expressions hold not only when B and P are whole numbers but also when they are decimal numbers. Case (2) seems harder to understand for students than case (1). In other words, with whole numbers, it could be easily understood that to determine the base amount means to determine the amount of one of P equal partitions. However, when the divisor is a decimal number, it is difficult to generalize this process of determining the per-unit quantity (the base quantity). Therefore it is important to teach this by using not only mathematical formula or expressions with words but number lines or diagrams, and by putting the problems in concrete situations so that students may find it easier to understand. Also, it is a good idea to write a multiplication expression first, and to use division to find the answer. Many students have hard time understanding the relationship between the divisor and the quotient when the divisor is smaller than 1. It is important to utilize things such as number lines and help students to be able to explain why the quotient is bigger than the divisor when the divisor is smaller than 1.

b. Multiplication and division of decimal numbers

Decimal numbers are represented using the same base-10 numeration system as whole numbers. Therefore, multiplication and division of decimal numbers can be carried out in the same manner as multiplication and division of whole numbers, if we pay attention to the place value of numbers and the position of the decimal points, and move the decimal points to see decimal numbers as whole numbers. Students should come to appreciate that the base-10 numeration system is useful this way in calculations with decimal numbers.

As for how to calculate multiplication and division of decimal numbers, we can think about the calculation process using the properties of calculations discussed in section c, below, to convert calculations with decimal numbers to calculations with whole numbers.

As for multiplication of decimal numbers, help students utilize the properties of multiplication—such as if the multiplier is multiplied by 10 the product is also multiplied by 10—so that they can think through multiplication of decimal numbers by converting it to whole number multiplication which they have learned before. For example, 12×4.3 can be thought of as $12 \times (4.3 \times 10) \div 10 = 12 \times 43 \div 10$.

As for the division of decimal numbers, help students utilize the property of calculation that "in the calculation of division, if both divisor and dividend are multiplied by the same number, the quotient remains the same" in order to be able to think about how to calculate with decimals. For example, $7.2 \div 2.4$ can be thought of as $(7.2 \times 10) \div (2.4 \times 10)$, hence $72 \div 24$.

In the case of division of decimal numbers with remainders, many students make mistakes about where to put the decimal point. When teaching, help students think about the meaning of the size of the remainder and the relationship between the quotient, divisor and remainder by using the fact that the remainder is smaller than the divisor and applying the formula (dividend) = (divisor) × (product) + (remainder).

c. Properties that hold for multiplication and division of decimal numbers

Help students understand that the properties that hold for the multiplication and division of whole numbers also hold for decimal numbers.

For example, if you calculate 30×2.5 and $30 \times 2+30 \times 0.5$ respectively, you get the same answer. Generally speaking, the distributive law $\bigcirc \times (\Box + \triangle) = \bigcirc \times \Box + \bigcirc \times \triangle$ holds for decimal numbers as well.

It is important to think about how to calculate or check the result of calculation by using these properties of calculation.

Mathematical Activities (1) a

Investigating and explaining the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines

The objective of this activity is for students to express mathematically what they think based on what they have already learned, and to enhance their thinking to something better and easier to understand. That is, to explain the meaning of multiplication and division of decimal numbers in words or using a number line based on the ideas already learned about multiplication and division.

Broaden the meaning of the multiplication of decimal numbers so that it can be used when the multiplier is a decimal number. For example, when thinking about the meaning of 120×2.5, students are expected to explain the meaning of multiplication by using a number line like the one below, or by expressing it in words such as "the amount corresponding to 2.5 when 120 is considered to be 1," or by using a formula or expressions with words.



Also, when doing an activity to explain the method of calculation, students can use a number line or properties of calculation. For example, when thinking about how to calculate 120×2.5 , if a property of calculation is used, students are to explain that idea with words such as "Multiply the multiplier 2.5 by 10 and make it 25. Calculate 120×25 , then divide the result by 10 to get the real result."

It is important to utilize the same kind of activities when teaching the meaning of division of decimal numbers and methods for their calculation.

A (4) Fractions

- (4) Students will deepen their understanding of fractions. They will understand the meaning of addition and subtraction of fractions with unlike denominators and be able to calculate accurately.
 - a. To be able to write decimal numbers as fractions and vice versa.
 - b. To understand that the result of dividing a whole number by another whole number may be expressed as a fraction.
 - c. To understand that when the numerator and the denominator are multiplied or divided by the same number, the size of a fraction remains the same.
 - d. To consider the size relationship of fractions and summarize the ways to compare fractions.

e. To explore ways to add and subtract fractions with unlike denominators and be able to calculate accurately.

Terms/Symbols (1) finding a common denominator, simplifying a fraction

In grade 5, students are expected to deepen their understanding of the meaning of fractions and how to represent them, and to think about how to add and subtract fractions with unlike denominators and become able to actually calculate them. Also, teach multiplication and division of fractions where the multiplier or divisor is a whole number.

Teach not only calculations with proper fractions but also calculations with improper fractions and with mixed numbers. In this case, avoid unnecessarily complicated calculations; rather, teach so that students can utilize the calculation of fractions in their daily life or future study.

a. Relationships between fractions and whole numbers or decimal numbers

In understanding the relationship among whole numbers, decimal numbers, and fractions, it is important for students to realize that whole numbers and fractions, and decimal numbers and fractions, are not different things, but are all numbers, even though they use different notations.

In general, when you want to represent a whole number by a fraction, if you assume *a* is a whole number, then $a = \frac{a}{1}$. But, the denominator of a fraction representing a whole number is not necessarily 1, and the value of the numerator is decided by the value of the denominator. For example, $2 = \frac{2}{1} = \frac{4}{2} = \dots$

When you want to convert decimal numbers into fractions, you can use 10, 100, or 1000 for denominators. For example, if you want to represent 0.13 using a fraction, since 0.13 is 13 if you use $\frac{1}{100}$ as a unit, you can represent it as $\frac{13}{100}$.

Also teach how to represent fractions using whole numbers or decimal numbers. For example, $\frac{1}{4}$ can be seen as $1 \div 4$ and can be represented as 0.25. It is important to help students understand that there are fractions that cannot be represented by whole numbers or finite decimal numbers. For example, $\frac{1}{3}$ =0.3333....

b. The results of division and fractions

It is always possible for the result of multiplication of two whole numbers to be represented by one whole number. In contrast, the result of division with two whole numbers cannot always be represented by one whole number or by one finite decimal number because there are cases where the quotient is not divisible even if the calculation proceeds into decimal numbers ($4 \div 3=1.33...$). By allowing the quotient of $a \div b$ (a and b

are both whole numbers and *b* is not 0) to be represented by a fraction, $\frac{a}{b}$, the result of division can always be represented by one number.

It is desirable to help students pay attention to the fact that quotients that are whole numbers (e.g., $6 \div 3=2$) or decimal numbers (e.g., $2 \div 4=0.5$) can also be represented by fractions. Also, it is important to efficiently employ the idea that $a \div b$ can be represented by $\frac{a}{b}$, or that $\frac{a}{b}$ can be interpreted as $a \div b$ when representing fractions using decimal numbers or when multiplying or dividing fractions.

c. Fractions representing the same size

A fraction whose numerator and denominator are multiplied or divided by the same number represents the same size as the original fraction. For example, $\frac{2}{3} = \frac{2 \times 3}{3 \times 2} = \frac{4}{6}$, and $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$. As shown, fractions have the property that many fractions can represent the same size. This is the same property that holds for division, as can be seen from $a \div b = \frac{a}{b}$. Consideration should be taken so that students can be aware of these kinds of properties when thinking about how to calculate with fractions.

Simplifying a fraction means to divide the numerator and the denominator of a fraction by a common divisor to make its denominator smaller. A simplified fraction represents the same size as the original fraction. Even when teaching the reduction of fractions, it is important to utilize a number line or a diagram.

In this way, it is important to enrich the students' sense of numbers through teaching different fractions that represent the same size.

d. Size and equivalence of fractions

As is discussed in c, there are many different fractions representing the same size. For example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, ... all represent the same size. The similarity and difference of size of several fractions with different denominators can be compared by converting those fractions to a common denominator. To convert fractions with different denominators to fractions with a common denominator is called *finding a common denominator*. The simplest way to convert two fractions with different denominators to a common denominator is to use the least common multiple as their common denominator.

When teaching "finding a common denominator," it is important that students understand its meaning and pay attention to the set of fractions of the same size. Teaching only the procedural aspect should be avoided.

e. Addition and subtraction of fractions with different denominators

If you convert fractions with different denominators to fractions with a common denominator, their addition and subtraction can be done in the same way as addition and subtraction of fractions with the same denominators. For example, $\frac{1}{2} + \frac{1}{3} = \frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Instead of simply teaching the procedure for finding a common denominator, it is important for students to think that by determining a common denominator they can see the calculation process as thinking about how many unit fractions there are. This idea of finding a common denominator is based on the fundamental principle of addition and subtraction: to add or subtract quantities, the unit of the quantities must be the same.

f. Multiplication and division of fractions

Here, $(fraction) \times (whole number)$ and $(fraction) \div (whole number)$ are taught. The meanings of $(fraction) \times (whole number)$ and $(fraction) \div (whole number)$ can be explained in the same way as multiplication and division of whole numbers. The meaning of multiplication can be thought of as adding the same number several times (repeated addition), or finding the corresponding compared quantity for the relative value when the base quantity and relative value are known. The meaning of division is the reverse of that of multiplication, and can be explained as finding the relative value or the base quantity.

When teaching multiplication and division, it is necessary to help students so that they willingly utilize the calculation of whole numbers or decimal numbers when they learn how to calculate fractions, and one should avoid making students memorize procedures.

B. Quantities and Measurements

B (1) Area of geometric figures

- (1) Students will be able to calculate the area of various figures.
 - a. To explore ways to calculate the area of triangles, parallelograms, rhombuses, and trapezoids.

Mathematical Activities (1)

b. Investigating and explaining the way of determining the area of triangles, parallelograms, rhombuses, and trapezoids, using concrete materials, words, numbers, mathematical expressions and diagrams.

The main objectives of grade 5 are for students to determine the area of fundamental geometric figures that are surrounded by straight lines by utilizing the methods for determining the area of rectangles and squares that they have previously learned, and to develop and apply new formulas for determining the area of new geometric figures. It is important to emphasize the process of thinking about how to determine areas or making formulas for finding areas based on previously-learned ideas and experiences.

a. Areas of triangles, parallelograms, rhombuses, and trapezoids

The main objectives here are to determine how to find the areas of triangles, parallelograms, rhombuses, and trapezoids based on the methods already learned for determining the areas of geometric figures, to explain the methods, to create formulas, and to cultivate the ability to think logically in the process. For example, students are expected to explain how to determine area based on the ideas below:

- (1) Move part of the geometric figure and transform it to an already-learned shape of equivalent area
- (2) Think about a shape as half of the area of an already-learned geometric figure
- (3) Decompose a shape into several already-learned geometric figures



By acquiring these ideas while learning how to determine the area of a triangle, students will become able to exercise ingenuity in determining area on their own. Moreover, students are expected to be able to determine the area of composite figures, made of basic figures such as triangles, and general quadrilaterals, by utilizing these ideas.



base. That is, if a certain side is selected as a base, the height is automatically defined, and no matter which side becomes the base, the area stays the same.

When teaching, it is not enough to always give students all the necessary measurements and let them use the formula to determine the area. In order to deepen their understanding of formulas, it is necessary also to provide situations where 5cm students need to think about which lengths they should measure to determine the area. Also, for students to be able to utilize formulas for finding area, it is important that students be able to choose the necessary information when measurements of many sides are given, such as in the diagram at right.



It is also important for students to deepen their understanding of the meaning of formulas for areas such as triangles through such activities as the following: to think about the change of the area when the base becomes two or three times as long as the original, while the height remains fixed; to determine the area of a triangle with decimal-number lengths in relation to calculations of decimal numbers in "A. Numbers and Calculations;" or the study of mathematical expressions and proportion in "D. Quantitative Relations." Concrete diagrams as well as formulas should be utilized in these activities.

Mathematical Activities (1) b

Investigate and explain ways to determine the area of triangles, parallelograms, rhombuses, and trapezoids using concrete objects, words, numbers, mathematical expressions, and diagrams

The objective of this activity is to foster the ability to think and explain mathematically through activities in which students think about and explain how to determine area. Another objective is for students to experience and understand that the study of determining area expands upon ideas that have already been learned. Yet another objective is for students to really feel the importance of mathematical thinking and a rich sense of geometric figures when creatively and expansively building up the study of area. These activities are for determining the area of triangles, parallelograms, rhombuses, and trapezoids. By repeating the activities, students can grow in their mathematical thinking ability, expressive ability, and disposition.

In the context of finding the area of special parallelogram (b)—after having found the area of parallelogram (a) using an equivalent-area transformation to transform it into a rectangle—"to think and to explain" means the following: explain that the area can be determined by transforming it into a rectangle (see (c)), or explain that the area can be determined based on a parallelogram (like (a)) whose area students already know how to find (see (d) and (e)).



Students can experience and understand how to determine the area of a trapezoid using the previously-learned formulas for the area of a parallelogram or the area of triangle, as shown in the diagrams below.



For (f), students can explain using concrete objects; e.g.: "Cut the trapezoid along the diagonal into two triangles. Then, determine the areas of those triangles." For (g), students can explain in the following way: "Use a congruent trapezoid, and combine those two trapezoids to get a parallelogram. Here, the base of the parallelogram is (upper base)+(lower base)." In (h) and (i), students can make an explanation such as: "When I moved a part of a trapezoid, I could make a shape we already learned. So I calculated using the formula for the area of that shape."

Through repeating these activities, students will understand, for example, that the idea for finding the area of a triangle using the equivalent-area transformation of decomposing the triangle into two parts by bisecting the height and forming a parallelogram can be used to determine the area of special triangles and trapezoids. By utilizing these ideas repeatedly, students will understand that these ideas are important in the study of area.



B (2) Units and measurement of volume

- (2) Students will understand the meaning and recognize the units for measuring volume; and they will be able to calculate volume.
 - a. To become aware of the units of volume cubic centimeter (cm^3) and cubic meter (m^3).
 - b. To explore ways to determine the volume of cubes and rectangular prisms.

By a grade 4, students have developed an understanding of measuring volume -mainly liquid volume -- based on a unit. The objectives of grade 5 are for students to be able to determine the volume of solids; to understand the meaning of units of volume; and to understand the meaning of the measurement of the volume of solids, i.e. that the volume of a solid can be expressed with a number if we determine a unit and talk about the number of units, similar to the method used for measuring area. Also, students are expected to understand that the volume of cubes and rectangular prisms can be determined by computation based on the measurements of the lengths of the sides that define the geometric figure, just like they did for finding areas of squares and rectangles, instead of using a tool to measure the volume directly.

a. Units of volume (cm³, m³) and its measurement

Analogous to determining area based on a unit size, here students are expected to understand that a unit of volume should be a solid figure where the space can be filled without any gaps. Moreover, students should understand that it is convenient to use a cube—whose volume can be determined by the length of one of its sides—as that solid figure; and they should understand that it is convenient if the size of the unit is fixed at 1 *cm* or 1 *m* along the side.

Moreover, students are expected to be aware of the inconvenience of using cubic centimeters (cm^3) when measuring the volume of a huge object, because the number can become too large to handle easily. In such cases, the bigger unit, cubic meter (m^3), should be introduced.

Also, consideration should be taken so that students enrich their sense of volume through activities determining the volume of cubes and rectangular prisms in their surroundings, or observing a cube whose actual volume is $1 m^3$. Moreover, the fact that the volume of a cube is 1 l if one of its sides is 10 cm should be touched upon.

b. Determining the volumes of cubes and rectangular prisms

Cubes and rectangular prisms can be made by piling up unit cubes whose side is 1 *cm* or 1 *m*. Consequently, students are to understand, from the analogy of determining the area of rectangles, that the volume of cubes or rectangular prisms can be determined by measuring their length, width, and height, and so obtain the formula (volume of rectangular prism) = (length)×(width)×(height). At the same time, it is important to help students understand that the first layer, where the unit volume is tiled without any gaps, can be represented by (length)×(width), and the number of layers is indicated by height.

Also, it is important to for students to foster a sense of volume through experiential activities where they actually determine the volume of cubes and rectangular prisms in their environment.

It is also important for students to deepen their understanding of the meaning of the formula for volume in relation to mathematical expressions or, as part of the study of proportional relationships in "D. Quantitative Relations," by thinking about the change in volume of a rectangular prism when the height gets twice, three times, four times, … greater while the length and the width remain fixed. This concept is utilized in determining the volume of prisms and cylinders in grade 6.

B (3) Measurement of quantities

(3) Students will understand the measurements of various quantities.a. To become aware of the mean of measured quantities.

By grade 4, students learned basic ideas of measurement, such as choosing appropriate measuring instruments and units depending on what would be measured and the size of the object to be measured; how to read tick marks; and how to express measured quantities. The objective of grade 5 is to help students to be able to express the results of measurements using appropriate numerical values by utilizing means.

Knowing an approximate size through estimation when measuring is important for planning how to measure, as it reduces the chance for measuring errors, and enables one to select a suitable unit and measuring instrument. It is important to help students understand that when you determine a measured value by reading tick marks on a measurement instrument, the result of the measurement is an approximate value, because when you measure, you round the smallest digit to the nearest tick mark, or deciding to divide the space between the smallest intervals into ten equal parts and decide on the smallest measurable digit by reading the tick marks on the instrument.

a. The mean of measured values

It is necessary for students to be made aware of the fact that there are always limits to measurement precision, and teaching should help students take this fact into consideration. Generally speaking, when there are several values as the result of measuring one thing, we may be able to get an appropriate value if we even out those values. So here the idea of using the mean of measured values is introduced. For example, if you measure some distance by pacing it off, you usually walk the distance several times and take the mean. To be able to calculate procedurally is not enough. It is necessary to help students understand the meaning of this. If there are outstanding or unexpected values, ask students to investigate the reason, and help them to be able to think about omitting those values to find the mean. For example, we may consider omitting the data for failed jumps—e.g. because the last step was not lined up well on the takeoff plank—when getting the mean of long jump data. Activities for obtaining the mean of measured values can be utilized not only in daily life but also in science class.

As for the calculation of the measured values, in relation to the teaching of an estimated calculation of product and quotient in "A. Numbers and Calculations," help students to aware that it is meaningless to get more digits than the original number of digits when finding a product or quotient. Teach students that unless there is a specific need, the number of the digits to be sought when finding a product or quotient is usually kept to that of the original numbers.

B (4) Ratio of two different kinds of quantities

- (4) Students will understand how to represent and compare quantities that are obtained as a ratio of two unlike quantities.
 - a. To become aware of per-unit quantities.

In grade 5, students will learn that, besides the quantities they have learned so far, there are quantities that are represented by the ratio of two quantities. In order to compare the yield of rice, one might consider the yield of rice per 10 *a*; in order to compare the density of people, one might compare the population per one square kilometer, which is often referred to as population density. These are examples of such kinds of quantities. The main objective is for students to understand this method of comparing and representing quantities, and to become able to use them.

When comparing, since this is the first time for students to compare quantities that do not have a characteristic of fundamental quantity, teach carefully so that they understand the meaning of comparing quantities that can be considered as ratios of two different kinds of quantities. First, like the population density example, help students understand that there are situations where they cannot make comparisons by paying attention to only one quantity, such as number of people, and moreover that there are quantities that cannot be quantified by the idea of measurement, i.e. counting the number of a unit quantity. In this case, it is important for students to discover the existence of such quantities, so teachers need to provide concrete situations and have students compare these quantities with quantities like weight or length. For example, the teacher needs to help students notice that one cannot determine whether a playground is crowded or not without considering both the area of the playground and the number of students playing on it, and compare and contrast such a situation with comparisons of length or weight.

Next, help students think about how they can compare and quantify the situations. In general, since there are two kinds of quantities involved, we can usually use the method of fixing one kind of quantity and compare the other quantity. When we use this way of thinking, we assume that two quantities are in proportional relationships. We also involve the idea of average. When teaching, it is necessary to help students pay attention to these points, and help them understand their meanings. For example, in the case of population density, two kinds of quantities are involved: population and area. Since the degree of congestion stays the same if an area gets twice, three times, or four times larger when the population gets twice, three times or four times bigger, help students notice that if we use this method and fix one quantity—for example the area—and compare the other quantity—the population—it is much easier to make comparisons. That is, when we want to compare the degree of crowdedness between 7 people in a 10 m^2 room and 10 people in a 15 m^2 room, if we make the area of the rooms 30 m^2 , the number of people in each room becomes 21 and 20 people respectively, so that we can come up with the idea of comparing by equalizing the size of the areas.

a. Size of per-unit quantities

The objective of this content is for students to understand that it is more efficient to use per-unit quantities when comparing quantities that involve a ratio of two different kinds of quantities, particularly when we compare three or more such quantities or wish to be able to make comparisons at any time. Moreover students need to become able to utilize the method of per-unit quantity to compare things.

When comparing the population density, it does not matter whether we consider the area as the unit quantity and compare the population per km^2 or consider the population as the unit quantity and compare the area per person. In general, however, in the case of population density, since it is more convenient if a big number corresponds to a high population density, we often use the area as the unit quantity and compare the population.

C. Geometric Figures

C (1) Properties of plane figures

- (1) Students will deepen their understanding of geometric figures by observing and composing geometric figures.
 - a. To recognize polygons and regular polygons.
 - b. To understand congruence of geometric figures.
 - c. To investigate and construct geometric figures by finding the properties of fundamental figures.
 - d. To understand the ratio of the circumference of a circle to its diameter (*pi*).

Mathematical Activities (1)

- c. Drawing and making congruent figures.
- d. Determining inductively the fact that the sum of the angles in a triangle is 180 degrees and explaining their thinking. Determining deductively the fact that the sum of angles in a quadrilateral is 360 degrees and explaining their thinking.

Remarks Concerning Content

(2) As for the content C-(1)-d, 3.14 is used as the ratio of the circumference of a circle to its diameter (*pi*).

In grade 5, teach polygons and regular polygons and help students deepen their understanding of plane figures.

a. Polygons and regular polygons

Polygons are the geometric figures surrounded by three or more straight lines. For example, a geometric figure surrounded by six straight lines is called a *hexagon*. Geometric figures whose sides are all equal length and whose angles have the same measure are called *regular polygons*. Equilateral triangles and squares are regular polygons.

Regular polygons have properties such as exactly fitting into a circle (they can be *inscribed* in a circle) and exactly fitting outside a circle (they can *circumscribe* a circle).

For example, when you connect the center of a circle and each vertex of a regular octagon inscribed in the circle by straight line, triangles are formed. Those triangles are isosceles triangles, all congruent. If you arrange six congruent equilateral triangles so that they share one vertex, you can form a regular hexagon. The geometric figure formed by cutting the circumference of a circle by its radius is a regular hexagon whose side is the same length as the radius of the circle.

In this way, you can examine the properties of regular polygons in connection with circles and the drawing of circles.



b. Congruency of figures

Students have experienced the fact that if they cut a square or an isosceles triangle in half, they can get two triangles of the same shape and size.

In grade 5, students are expected to understand the congruency of geometric figures. When two geometric figures overlap exactly, that is, when two shapes are the same shape and size, they are called congruent. When two geometric figures are congruent, their corresponding side lengths and corresponding angle sizes are the same. It is important for students to be able to determine and verify the properties of geometric figures through activities such as finding, drawing, and constructing congruent geometric figures.

It is also important for students to be able to identify the corresponding sides and angles of two congruent geometric figures regardless of their orientations, even when they are slid, rotated, or flipped.

The diagrams below show that when dividing a parallelogram into two triangles along a diagonal, you get congruent geometric figures. Building upon this observation, students can also investigate combining two congruent triangles to form a parallelogram.



c. Properties of geometric figures

In grade 5, help students to identify the properties of geometric figures and teach them to use the properties to examine and compose geometric figures.

There are things that always hold for a certain geometric figure. Those things are called *properties* of geometric figures. For example, if you add the measures of three angles of any triangle, the result is 180°. This is a property of triangles. This property can be explained by thinking inductively. To think inductively means to find a general phenomenon based on common features of several concrete examples.

As for quadrilaterals, if you add the sizes of four angles of any quadrilateral, the result is 360°. This is a property of quadrilaterals. This property can be explained by thinking deductively, using a property of triangles (i.e. the property that if you add the sizes of three angles of any triangle, the result is 180°.)

In addition to these properties, students are expected to find properties of geometric figures they have already learned, and be able to explore and construct shapes using those properties. In order to find the properties of geometric figures, it is necessary to incorporate activities such as composing and decomposing geometric figures. It is also important to help students be interested in thinking logically and to appreciate its value.

d. The ratio of the circumference of a circle to its diameter (pi)

Students learned about the center, radius, and diameter of a circle in grade 3. In grade 5, they will learn the meaning of the ratio of the circumference of a circle to its diameter (*pi*).

Help students be aware that there might be a relationship between the diameter and circumference of a circle, and to try to help them have foresight of how many times the diameter is the circumference. For example, if students use a regular hexagon inscribed in a circle and a square circumscribed around



it, they can see that the circumference is longer than three times the diameter (six times the radius) and shorter than four times the diameter. If students actually measure the diameters and circumferences of several circles and think inductively, they can find that the ratio of the circumference to the diameter of a circle remains the same regardless of the size of a circle. This ratio of the circumference to the diameter of a circle is called *pi*. By teaching this, help students understand the relationships between diameter, circumference, and *pi*, such as the students can calculate the circumference from the diameter and the diameter from the circumference.

Also, note (2) of "Remarks Concerning Content" states that "3.14 should be used as the value of pi."

Mathematical Activities (1) c

Activities such as drawing and constructing congruent shapes

The objective of these activities is to help students understand congruency by focusing on the conditions that are necessary for drawing or constructing congruent figures, through activities such as overlapping congruent shapes and tracing them, and drawing and constructing congruent shapes using the idea of correspondence.

It is important to help students notice that the following conditions are necessary for drawing and composing congruent triangles through activities such as drawing and constructing congruent figures.



Three corresponding sides are equal length



Two corresponding sides and the angle between them are equal length and size



One corresponding side and the two adjacent angles are equal length and sizes

Also, through activities where students verify that the figures they draw or compose are congruent, or that the figures satisfy certain conditions, students acquire the disposition to explain based on solid reasoning.

Mathematical Activities (1) d

Determining inductively that the sum of the angles in a triangle is 180 degrees an explaining their thinking. Determining deductively the fact that the sum of the angles in a quadrilateral is 360 degrees and explaining their thinking

The objective of this activity is for students to think about and explain the sum of the three angles in a triangle, and the four angles in a quadrilateral.

To think inductively is to find common and general facts among several concrete examples. The activity here is for students to think about and explain why the sum of the three angles of a triangle is 180 degrees through examining various triangles.

The methods for examining the sum of three angles of a triangle is 180 degrees are: tiling the congruent triangles; measuring angles using a protractor; gathering together the three angles, etc. Through these activities, help students discover with surprise that the sum of the three angles of any triangle is 180 degrees and appreciate its beauty.

To think deductively is to explain that something new is correct based on something already proved to be true. The activity here is to think about and explain that the sum of the four angles of a quadrilateral is 360 degrees based on the fact that the sum of the three angles of a triangle is 180 degrees.

There are two representative methods of thinking and explaining this fact deductively: (1) divide a quadrilateral into two triangles along a diagonal, and, based on the fact that the sum of three angles of a triangle is 180 degrees, deduce 360 degrees by multiplying 180 degrees by two; (2) add point E to the inside of a quadrilateral, to divide the inside of it into four triangles by connecting point E with each vertex, to multiply 180 degrees by four based on the fact that the sum of three angles of a triangle is 180 degrees, and then to subtract 360 degrees which is the sum of the angles around point E, to get 360 degrees. By utilizing these ideas, help students become interested in thinking logically and notice its value through thinking and explaining deductively.



C (2) Properties of solid figures

(2) Students will understand solid figures through observing and composing solid figures.

a. To recognize prisms and cylinders.

<u>Terms/Symbols</u> base, lateral surface

Remarks Concerning Content

(3) As for content C-(2)-a, drawing sketches and nets of shapes should be taught.

As for solid figures, in grade 4, students learned the properties of cubes and rectangular prisms by focusing on the number of their constituents such as sides, faces, and vertices; the shapes of faces; and parallel or perpendicular relations of faces or edges. In grade 5, prisms and cylinders are introduced as solid figures. The objectives here are for students to learn how to draw solid figures on a plane, to imagine the shape of solid figures from the drawings on a plane, and to enrich their sense of space.

Base and lateral surface are mentioned in "Terms/Symbols." It is important for students to be able to explain and express using these terms.

a. Prisms and cylinders

Prisms and *cylinders* are solid figures composed of *bases* and *lateral surfaces*. *Prisms* are solid figures whose bases are polygons and whose lateral surfaces are rectangles. For example, a prism whose base is a triangle is called a *triangular prism* and a prism whose base is a quadrilateral is called a *quadrilateral prism*. Students are expected to understand that cubes and rectangular prisms are prisms. *Cylinders* are solids whose bases are circles.

Through activities such as observing prisms and cylinders, students are expected to recognize the number or shape of their components—such as vertices, edges, and faces—and the parallel and perpendicular relationships between edges, between edges and faces, and between faces.

Note (3) of "Remarks Concerning Content" indicates that drawing sketches and nets of shapes should be taught. Through drawing sketches and nets of shapes, help students examine connections and positional relationships between edge and edge, edge and face, and face and face. Also, through the activity of drawing nets of solid figures and constructing those figures, students are expected to deepen their understanding of prisms and cylinders, and enrich their sense of space.

D. Quantitative Relations

D (1) Relationships between two quantities that vary together

(1) Students will use tables to consider the relationships between two quantities as the quantities vary simultaneously.

Terms/Symbols		
proportionality		

By the end of grade 4, students learned the relationships between two quantities that vary together in situations involving addition, subtraction, multiplication, and division. The objectives of grade 5 are, on top of that, for students to enhance their ability to examine two quantities that vary together to understand proportional relationships, and to develop their functional thinking.

When teaching, focus on using tables. It is important for students to become able to utilize tables by fully carrying out activities of making and interpreting tables. For example, when teaching content related to multiplication and division of decimal numbers, formulas for the area of triangles and quadrilaterals, and percentage and ratio, help students investigate two co-varying quantities using tables.

It is important to deepen the way of viewing quantitative relations through interpreting the characteristics of how quantities correspond and how they vary by utilizing tables.

a. Proportional relationships in simple cases

In grade 5, proportional relationships of two quantities as they vary together in simple cases are taught. "Simple cases" means, for example, relationships between two quantities whereby if one increases two, three times, four times, ... as much as the original quantity, the other quantity increases the same way, and the change can be seen from a table representing those changes. A concrete example would be something like: "the relationship between the number of the steps and the total height of the staircase when the height of one step of the staircase is 15 centimeters", or "the relationship between the area of a rectangle when the length is fixed at 6 centimeters." While teaching students to examine the changes in quantities while filling in the table through these activities, students will be able to identify the characteristics of how the two quantities correspond or how they change.

When teaching these points, help students become aware that they are closely related to multiplicative situations that they have seen previously. Also, help students verbalize the characteristics or rules they find—for example, "if the width becomes twice, three times, four times, … as long as the original, the area will become twice, three times, four times, … as big as the original one"—so that they can capture these characteristics and rules appropriately.

D (2) Mathematical expressions representing quantitative relations

(2) Students will deepen their understanding of mathematical expressions that represent quantitative relationships. They will be able to recognize the correspondence between two quantities and the aspect of variation in a quantitative relationship that can be represented by a simple mathematical expression. In grade 4, expressions that represent quantitative relations such as formulas or expressions with \Box and \triangle were taught. Building on these understandings from prior grades, the objectives of grade 5 are for students to examine quantitative relations represented by expressions such as $\Box=2+\triangle$, $\Box=2\times\triangle$, or $\Box=3\times\triangle+1$, and to deepen their understanding of the mathematical expressions that represent quantitative relations.

To be able to interpret the meaning of mathematical expressions, teacher need to provide ample opportunities to examine the characteristics of how two quantities that vary simultaneously correspond or change by examining tables, and ample opportunities to represent the relationship between two quantities with mathematical expressions using words. Also, to deepen their understanding of the meaning of mathematical expressions, it is important to broaden students' functional thinking.

Teach so that the relations represented by mathematical formulas apply not only to whole numbers but also to decimal numbers, and deepen students' understanding of the role that mathematical expressions play in representing quantitative relations and rules concisely and generally.

D (3) Percentage

(3) Students will understand percentage.

Terms/Symbols

%

Remarks Concerning Content

(4) As for content D-(3), representing ratio using *buai* [the ratio based on 10 percent in Japanese] should be taught.

When investigating data numerically, sometimes we use difference to compare two quantities and sometimes we use ratio. When we consider the relationship between the whole and part of the data, or among parts of data, we often use ratio.

In grade 4, students experienced how to set the size of a base quantity as 1 and to represent the relative value of a compared quantity using decimal numbers. The objective of grade 5 is to understand percentage and become able to use it.

Percentage is a way to represent the relative value of a compared quantity using whole numbers as much as possible by setting the size of the base quantity at 100. It is important to help students recognize the merits of representing ratio using whole numbers.

In daily life, percentage is used to describe a settled incident such as "The ratio of absenteeism was 15%" or "I purchased it for 20% off the regular price." It is also used to

describe an uncertain incident; for instance, the weather report might say, "Tomorrow's chance of precipitation is 20%." Consideration should be taken to accommodate activities such as finding occasions in daily life where percentage is used so that students become aware that mathematics is used in various parts of our daily lives.

Note (4) of "Remarks Concerning Content" of grade 5, states that, "Representing ratio using *buai* [the ratio based on 10 percent] should be taught." Concerning the meaning of *buai*, which we teach here, teachers should touch upon the idea that one can consider the compared quantity as 10, and express the relative value as *wari* as is done with percentage. Also help students recognize that *buai*, like percentage, is a useful way to represent ratios in everyday life.

D (4) Pie charts and percentage bar graphs

(4) Students will be able to gather, sort, and organize data according to their purpose, to represent data in pie charts and percentage bar graphs, and to investigate the features of data.

<u>Mathematical Activities</u> (1)

e. Selecting and using appropriate tables and graphs to display data purposefully.

The objective of grade 5 is to help students understand that there are phenomena whose characteristics are easy to grasp when we compare the whole and the part or compare part to part, and to help them make and interpret pie charts or percentage bar graphs that represent these relationships.

When teaching pie charts and percentage bar graphs, one should teach not only how to draw them but also how to interpret them in relation to percentage. It is important to take advantage of the characteristics of the representation known as a graph, and to foster a statistical viewpoint through these activities. As for pie charts, circular graphing paper with 10 or 100 equally divided tick marks on the circle should be used.

Mathematical Activities (1) e

Selecting and using appropriate tables and graphs to display data purposefully

The objective of this activity is for students to become able to utilize tables and graphs by selecting more suitable tables or graphs and using them to solve problems purposefully through representing, interpretating, and judging. Another goal of this activity is for students to enjoy learning mathematics, and for them to really feel and experience mathematics.

In grade 5, students learn about two-dimensional tables for tables, and bar graphs, broken-line graphs, pie charts, and percentage bar graphs. It is important to provide opportunities for students to explain the characteristics of data or express what they want to say using the knowledge and skills with tables and graphs that they have acquired.

Make the purpose of the activity clear such as: "show the size of quantities," "show the change of quantities," or "show the ratio of quantities," and help students select the appropriate table or graph for the purpose. Then, carefully consider the choice of table and graph, asking students to think about what categorization or items need to be indicated, how to place tick marks, or if multiple graphs should be combined. Also, plenty of activities such as interpreting these things in relation to one another or judging them should be carried out.

Mathematical Activities

- For the contents of "A: Numbers and Calculations", "B: Quantities and Measurement", "C: Geometric Figures", and "D: Quantitative Relations", instruction should incorporate mathematical activities such as those listed below.
 - a. Investigating and explaining the meaning and the ways of calculating with decimal numbers, using words, numbers, mathematical expressions, diagrams, and number lines.
 - b. Investigating and explaining the way of determining the area of triangles, parallelograms, rhombuses, and trapezoids, using concrete materials, words, numbers, mathematical expressions, and diagrams.
 - c. Drawing and making congruent figures.
 - d. Determining inductively the fact that the sum of the measures of the angles in a triangle is 180 degrees and explaining their thinking. Determining deductively the fact that the sum of the measures of the angles in a quadrilateral is 360 degrees and explain their thinking.
 - e. Selecting and using appropriate tables and graphs to display data purposefully.

Terms/Symbols

greatest common divisor, least common multiple, finding a common denominator, simplifying a fraction, base, lateral surface, percentage, %

6. The Content of Grade 6

A. Numbers and Calculations

A (1) Multiplication and division of fractions

- (1) Students will understand multiplication and division of fractions, and be able to use them appropriately.
 - a. To understand the meaning of multiplying and dividing by fractions based on multiplication and division by whole numbers and decimal numbers.
 - b. To explore ways to calculate multiplication and division of fractions, and be able to calculate accurately.
 - c. To understand that the same properties of operations that are true with whole numbers still apply when numbers become fractions.

Mathematical Activities (1)

a. Investigating and explaining the ways of carrying out fraction computations using words, numbers, mathematical expressions, diagrams and number lines.

Remarks Concerning Content

(1) As for the content A-(1), looking at division as multiplication by using reciprocal numbers, and combining multiplication and division involving several numbers (including whole numbers and decimal numbers) in a calculation of fractions should be taught.

The four arithmetic operations of whole numbers and decimal numbers have been taught by grade 5. As for fractions, addition and subtraction of fractions with like and unlike denominators and multiplication and division of fractions in which the multiplier or divisor is a whole number have been taught as well.

The main objective of grade 6 is for students to think about how to multiply and divide in cases where the multiplier or divisor is a fraction and actually be able to perform the calculations.

As for calculating fractions, teach by starting with proper fractions, and also include improper fractions and mixed numbers. Avoid teaching unnecessarily complicated calculations. It is important to teach in a way that students can value and utilize the calculations of fractions in their everyday life and future studies. Help students become aware of the fact that multiplication and division of fractions are much easier to carry out if you use improper fractions and not mixed numbers.

a. The meaning of multiplication and division when the multiplier or divisor is a fraction

In grade 5, the meaning of multiplication with a whole number multiplier was taught. This was explained as adding the same number many times (repeated addition), or finding the total size from the unit size and the ratio on which it is based. Division of fractions with whole number divisors is considered to be the inverse of multiplication.

In grade 6, based on the idea of calculation of whole numbers and decimal numbers taught so far, students are expected to understand multiplication and division when the multiplier or divisor is a fraction. The meaning of multiplication can be expressed as such: If B is the base quantity, P is the relative value, and A is the corresponding quantity that has the specified relative size with respect to B, their relationship can be represented by $B \times P = A$. The meaning of division can be thought of as the inverse of multiplication: finding the relative value or finding the base quantity.

b. Multiplication and division of fractions

Students are expected to think about and be able to carry out multiplication and division with fractions, based on the multiplication and division of whole numbers or decimal numbers, or multiplication and division of fractions where the multiplier or divisor is a whole number. A multiplication or division of fractions can be carried out by changing it to a multiplication or division that was discussed in a previous grade by applying the properties discussed in the next section (c). Students can use the number line or diagrams to think about how to perform those calculations. This is discussed in Mathematical Activities (1)-a.

For reciprocal numbers discussed in note (1) of "Remarks Concerning Content," the fact that division can be converted to multiplication by the reciprocal of the divisor is taught. For example, since the reciprocal number of the divisor in $\frac{2}{5} \div \frac{3}{4}$ is $\frac{4}{3}$, this division

can be rewritten as $\frac{2}{5} \times \frac{4}{3}$.

Representing multiplication and division of whole numbers and decimal numbers using fractions is also taught. For example, $5 \div 2 \times 0.3 = \frac{5}{1} \times \frac{1}{2} \times \frac{3}{10} = \frac{5 \times 1 \times 3}{1 \times 2 \times 10}$.

c. Properties of multiplication and division with fractions

Students will examine properties of multiplication and division that apply to whole numbers, to decimal numbers, and to fractions. The properties to be investigated are not limited to commutative, associative and distributive properties. In addition, students will examine the property of multiplication that when the multiplier increases by 2, 3... times, the product increases 2,3...times, and the property of division, that when the dividend and the divisor are multiplied or divided by the same number, the quotient does not change.

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For example, when dividing fractions, we can utilize the properties of division and think

of $\frac{2}{5} \div \frac{3}{4}$ as $\left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right) = \left(\frac{2}{5} \times 4\right) \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$. We can also think of it as: $\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{4}{3}\right) = \frac{2}{5} \times \frac{4}{3}$.

It is important to cultivate mathematical thinking such as logical thinking through these teachings.

Mathematical Activities (1) a

Investigating and explaining the ways of carrying out fraction computations using words, numbers, mathematical expressions, diagrams and number lines

An objective of this activity is to enhance students' ability to represent ideas mathematically based on their prior learning. Another objective is to foster the attitude to think logically with clear rationale. This means to think about and explain the meaning and the procedures of multiplication and division of fractions using their previous learning about the meaning of multiplication and division of fractions by whole numbers.

For example, when solving a problem such as, "There is a 1 meter rod that weighs $\frac{3}{4}$

kilograms. How many kilograms is $\frac{2}{3}$ meter of this rod?" students are expected to

express this problem—using words such as, "If we see $\frac{3}{4}$ kilogram as 1, and we want to

find $\frac{2}{3}$ of this 1"; using a mathematical expression with words such as "weight of 1 meter" × "length of the rod" = "weight of the rod"; or using a number line—to see that the answer can be calculated by $\frac{3}{4} \times \frac{2}{3}$.



Students will express their thoughts and ideas using numbers, mathematical expressions, or diagrams, based on their prior learning.

Students can also explain how to perform calculations using a number line or the properties of the four arithmetic operations. Here, students will explain that the answer is given by $\frac{3}{4} \times 2 \div 3$ or $\frac{3}{4} \div 3 \times 2$.



Students can also explain how to determine the weight of $\frac{2}{3}$ meter by seeing an actual rod. Looking at a rod that is 1 meter long and weighs $\frac{3}{4}$ kilograms, like the one in the diagram at left, students can explain that if the rod is cut into three equal parts, two of them will correspond to the weight of $\frac{2}{3}$ meter of the rod. This can be

represented by the mathematical expression: $\frac{3}{4} \div 3 \times 2$.

Here, again, students are further developing their attitude of thinking logically with clear rationale, as they base their reasoning on prior learning, i.e., (fraction) ÷ (whole number) and (fraction)×(whole number).

A (2) Consolidating computation skills with decimal numbers and fractions

(2) Students will extend their ability to use computations with decimal numbers and fractions in appropriate situations by further consolidating their computation skills.

The four arithmetic operations of decimal numbers and fractions are taught from grade 3 through grade 6. In grade 6, students are expected to consolidate the computational skills of the four arithmetic operations of decimal numbers and fractions, and to extend their ability to use them.

Computational skills involve understanding the meaning of computation and thinking about how to perform calculations. Computational ability involves mastering the basic and fundamental computational skills and utilizing those skills in life and learning.

B. Quantities and Measurements

B (1) Approximate shapes and estimated area

(1) Students will be able to estimate the area of shapes in their surroundings by approximating them with familiar figures.

The objective of grade 6 is to help students master measurements and be able to measure efficiently according to a purpose, using estimation and other techniques, by actually measuring a variety of objects.

By having students actually measure a variety of objects, the objective of grade six is to help them master measurements and be able to measure efficiently according to a purpose using estimation and other techniques.

When measuring the area or volume of an object around them, students may notice that the object does not necessarily have a basic shape. In such a case, they may consider it as a triangle or a rectangle, or divide it into several basic shapes. If the object they want to measure is a solid shape, they may consider it as a cube or a rectangular prism, or divide it into those shapes. Approximation plays an important role here. Students need to enrich their experiences with these processes by thinking about not only regular shapes but also about objects around them.

When calculating with measured values, make sure students understand that using too many digits is meaningless and help students calculate appropriately according to the objectives.

B (2) Area of a circle

(2) Students will be able to determine the area of figurers by calculation.a. To explore ways to determine the area of circles.

Remarks Concerning Content

(2) As for content B-(2)-a, 3.14 is used as a ratio of circumference of a circle to its diameter (*pi*).

By the completion of grade 5, students have learned how to determine the area of shapes such as triangles and quadrilaterals that are surrounded by straight lines. Students also learned that the circumference of a circle is represented by (diameter) \times *pi*.

In grade 6, students are taught how to think about determining the area of a circle—a geometric figure surrounded by a curve. When thinking about how to determine the area of a circle, it is important to first estimate the area. Looking at the diagram at right, it is clear that the area of a circle is somewhere between 2 and 4 times as large as the area of a square whose side is the length of the radius of the circle.

There are several ways of thinking about how to determine area. For example,:

(1) Draw a circle on a grid paper, count the squares in the circle, and determine the area.

The diagram at right shows a circle with a radius of 10 centimeters drawn on 1 centimeter grid paper. (The diagram

shows only $\frac{1}{4}$ of the actual circle.)





(2) As shown at right, equally divide a circle, rearrange the resulting shapes, make a parallelogram-like shape, and determine the area of a circle. In this case,



the more finely you divide a circle, the closer the base of the parallelogramlike shape gets to half of the circumference and the closer the height gets to the radius of the circle. Hence, the area of a circle can be represented by the following expressions:

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(area of circle) = (area of parallelogram) = (radius) \times (circumference) \div 2
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If you use (circumference) = (diameter) \times *pi*, then:

(area of circle) = (radius) × (diameter) × $pi \div 2$ = (radius) × (radius) × pi

Note (2) of "Remarks Concerning Content" says, "3.14 is used for *pi*." This is because if one uses 3.14, it is sufficient for general use in daily life and studies.

B (3) Volume of prisms and cylinders

- (3) Students will be able to determine the volume of figures by calculation.
 - a. To explore ways to determine the volume of prisms and cylinders.

Students were previously taught about the volume of cubes and rectangular prisms. They also learned prisms and cylinders as basic solid shapes in grade 5. The base of prisms and cylinders are triangles, quadrilaterals, circles, etc. They learned how to determine area of triangles and quadrilateral in grade 5, and how to determine area of circles in grade 6. The objective of grade 6 is for students to be able to determine the volume of prisms and cylinders. It is also necessary to consider deepening students' understanding of prisms and cylinders as geometric figures.

a. Determining volume of prisms and cylinders

The main objective of the study of prisms and cylinders here is for the students to understand that the volume of these shapes can be determined by calculation based on the ways of determining the volume of cubes and rectangular prisms taught in grade 5.

When determining the volume of cubes and rectangular prisms, first we thought about the volume of solid shapes whose height is 1 centimeter, then we multiplied that volume by the number corresponding to the height, and thereby derived the formula for volume. We can use this method to determine the volume of prisms and cylinders.

By analogy to the formula for determining volume of rectangular prisms, we can derive the formula for determining the volume of prisms and cylinders. First, we define that (length) \times (width) in the formula for rectangular prisms corresponds to (base area):

 $\begin{pmatrix} volume of a \\ rectangular prism \end{pmatrix}$ = length × width × height = (base area) × height

Based of this, students are expected to understand that the formulas for determining the volume of prisms and cylinders can be generalized as:

 $\begin{pmatrix} volume of prisms \\ and cylinders \end{pmatrix} = (base area) \times height$

B (4) Speed

(4) Students will understand and determine speed.

In grade 5, the ratio of two quantities of different types, such as crowdedness and population density, was taught.

In grade 6, speed is taught as the ratio of two different quantities.

For speed, students have heard people say fast or slow referring to the speed of someone running fast or some vehicle moving.

Two quantities—the distance to move and the time needed to move—are necessary to express speed as a quantity. If we consider speed as an amount of distance to move during a unit time, it can be expressed as (speed) = (distance) \div (time). For example, 60 kilometers per hour means the speed for moving 60 kilometers in one hour.

In our daily life, speed is sometimes thought of as the time required to move a particular distance. For example, in a 100-meter sprint, speed is indicated by the time required to run 100 meters. The shorter the time, the faster the speed.

If we think of speed as a distance to cover in a unit time, the faster the speed is, the larger the value becomes. If we think of speed as time spent moving a unit distance, the faster the speed is, the smaller the value becomes.

Since speed can be expressed by the mathematical expression (speed) = (distance) \div (time), speed can be determined by distance and time. Also, distance can be determined by speed and time, and time can be determined by speed and distance. It is important to connect speed with actual situations so that students can utilize it in their daily life and study.

B (5) System of metric units

(5) Students will understand the system of metric units.

<u>Mathematical Activities</u> (1)

b. Identifying units used in their surroundings and exploring how they relate to the units they studied previously.

By the end of grade 5, students understood the meaning of quantities such as length, weight, area, volume, time, and angle, and became able to measure them. The objectives of grade 6 are to summarize the metric system and its units, for students to deepen their understanding of it, and for them to be able to use metric units efficiently in measuring.

The characteristics of the metric system are that the units follow the structure of the base-10 numeration system, and that derived units are formed based on the basic units.

In the metric system, units are made by adding prefixes to the basic units, as in the following examples:

Mili (m)	Centi (c)	deci (d)		Deca (da)	Hect (h)	kilo (k)
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10 times	100 times	1000 times

Though there are units in this chart that are seldom used in our country [Japan], the metric system can be summarized based on these prefixes.

For units of area and volume, derived units are based on the unit of length and they are summarized in the chart below:

Unit of length	1 <i>cm</i>	(10 <i>cm</i>)	1 <i>m</i>	(10 <i>m</i>)	(100 m)	1 km
Unit of area	1 <i>cm</i> ²	(100 cm ²)	1 m ²	$(100 m^2)$ 1 a	(100 a) 1 ha	1 km ²
Unit of volume	1 cm ³ 1 ml	(1000 cm ³) 1 l	1 m ³ 1 kl			

Mathematical Activities (1) b

Identifying units used in their surroundings and exploring how they relate to the units they studied previously

The objective of this activity is to experience and understand the value of the metric system in that it enables us to guess the size of new units by analogy, utilizing what was learned about the workings of units of the metric system.

In the daily life, if one looks around enough, many units can be seen in students' surroundings. Milligram and centiliter are used to represent the quantity of drinks. Kiloliter is used to represent the amount of tap water consumed or the capacity of a tanker truck. These units are made by adding prefixes m (*milli-*) c (*centi-*) and k (*kilo-*) to the basic unit. So students are expected to explore around to find these units, then investigate the type of units they find as well as the relationship between these new units and the ones they have already learned. For example, if a student finds "100 *mg*" in the ingredients section on a box of medicine, he will understand this is the unit of weight since *g* (gram) is used. Also, he will understand from *m* (*milli-*) that 1 *mg*

(milligram) is $\frac{1}{1000}$ of 1 gram.

Here, it is important to foster the attitude in students to willingly examine the relationship between the units they find and units they learned in their daily life based on their interests.

C. Geometric Figures

C (1) Reduced and enlarged drawings, and symmetric figures

- (1) Students will deepen their understanding of geometric figures through observing and composing geometric figures.
 - a. To understand scale drawings.
 - b. To understand symmetric figures.

<u>Mathematical Activities</u> (1)

c. Identifying scaled drawings and symmetric figures in their surroundings.

<u>Terms/symbols</u> line symmetry, point symmetry

The objectives of grade 6 are to learn a new way of viewing geometric figures, and to use that new viewpoint to summarize plane figures students have already learned. The viewpoints to be used in summarizing are scaling and symmetry. Students are expected to deepen their understanding of geometric figures from these viewpoints and to enrich their geometry sense.

Line symmetry and point symmetry are terms listed in <u>Terms/Symbols</u>. It is important for students to be able to explain or represent figures using these terms.

a. Reduced and enlarged drawings

In grade 5, students learned about congruency and considered geometric figures from this viewpoint. In grade 6, reduced and enlarged drawings are taught. The objective of this grade is to enrich students' experience so it will provide the foundation for understanding similarity, and to help them become able to draw and interpret scaled drawings appropriately, according to a purpose.

When the shape and the size of two figures are the same, these two figures are said to be congruent. *Reduced or enlarged drawings (scaled drawings)* have the same shape regardless of their size. The measures of corresponding angles of scaled figures are equal, and the ratio of the lengths of the corresponding sides of scaled figures is constant.

When actually making scaled drawings, sometimes we scale vertically and horizontally using grid paper, and sometimes we scale the lengths of the sides or diagonals from one vertex, as shown in the figure below.



Students are expected to understand the meaning and characteristics of scaled drawing by actually drawing these kinds of figures.

b. Symmetric figures

Students are expected to consider geometric figures they previously learned from a new perspective of symmetry, and to deepen their understanding of geometric shapes.

For symmetry, we examine a shape from the viewpoints of line symmetry and point symmetry. *Line-symmetric* figures are shapes that overlap exactly when folded along a certain line. The folding line perpendicularly bisects all segments that connect corresponding points of the figure. *Point-symmetric* figures are figures that exactly overlap when rotated 180 degrees around a point. All segments that connect the corresponding points of a figure run through the center of symmetry and are bisected by this point.

It is important to understand the meaning of line symmetry and point symmetry through such activities as observation and drawing—including using symmetry to fill in missing parts of a figure—and to deepen the ways of viewing geometric figures by

distinguishing line-symmetric figures, point-symmetric figures, and figures that have both line and point symmetry.

When looking at triangles from the viewpoint of line symmetry, one can grasp that isosceles triangles and equilateral triangles are line-symmetric figures. When quadrilaterals are examined from the viewpoint of point symmetry, parallelograms, rhombi, rectangles, and squares are point-symmetric figures.

Students began to learn about symmetry in lower grades when they studied geometric figures. For example, when they learned about isosceles triangles in grade 3, they learned that an isosceles triangle exactly overlaps when folded by the perpendicular bisector of the base. In this way, students focused on symmetry through





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concrete manipulation. In grade 6, students are taught to focus on the beauty of balance, stability, and general beauty of geometric figures through activities such as observation and drawing.

Mathematical Activities (1) c Identifying reduced and enlarged drawings and symmetric figures in their surroundings

The objective of this activity is to have students become aware of the fact that reduced and enlarged drawings are utilized in various aspects of our daily life and to foster a disposition to willingly utilize them.



For example, help students become aware that scaled images are used in many aspects of our daily life—copy machines, maps, blueprints, microscope images, photos, and movies, etc.—and help them find scaled figures from the surroundings such as floors or walls tiled with congruent figures. Or have students determine the height of a tree by measuring not the actual tree, which is not feasible, but by measuring its shadow, which is easier to measure. Students can also determine actual distance from a map by calculation, utilizing its scale factor; for example, a ratio of 1:100 means 1 centimeter on a map represents 1 meter in reality. Another example is to have students calculate the actual length of the pool from a blueprint of a pool with a specific scale factor. Through these activities that illustrate their practical applications, foster in students a willingness to utilize scaled drawings in their daily life.

With respect to symmetry, the objective is for students to experience and understand that symmetric figures are used in their daily life by finding them in their surroundings. Symmetric figures like the ones in tiled figures and tiled patterns can be found in the beauty of balanced figures in everyday life. Symmetric figures can be found anywhere, such as in plants and animals, accessories, patterns, map symbols or prefecture marks. It is important to utilize activities involving finding these figures.

D. Quantitative Relations

D (1) Ratio

:

(1) Students will understand ratio.

Terms/Symbols

When comparing the size of two quantities and representing their proportions, we sometimes use a pair of simple whole numbers to represent this without using a unit quantity. This is called *ratio*. Students learned the foundation of ratio when they learned multiplication, fractions, and proportional relationships.

In grade 6, the representation of ratio in the form of *a*:*b* is introduced, and students will understand ratio.

When teaching, consideration should be taken so that students understand when ratios are equal and what the equality of ratios means by examining concrete situations. For example, suppose you mixed two different kinds of liquid, three cups for one and five cups for the other. If you want to mix these two kinds of liquid to get a liquid of the same concentration, the ratio of the two kinds of liquid should stay the same—such as 6 cups for one and 10 cups for the other or 9 cups for one and 15 cups for the other. From this fact, help students to understand that 3:5 is equal to 6:10, 9:15 or 1.5:2.5..

Ratio is used in various aspects of our daily life. When teaching, use ingenuity and incorporate activities such as finding places where ratio is used in daily life or utilizing ratio to handle things. Also, since ratio is related to proportional relationships, inverse proportional relationships, and scaled drawing, it is necessary to endeavor that the study of each of these topics deepens students' understanding of the others.

D (2) Proportional Relationships

- (2) Students will be able to consider the relationship between two quantities varying simultaneously.
 - a. To understand proportional relationships and to investigate their characteristics by using mathematical expressions, tables, and graphs.
 - c. To become aware of inversely proportional relationships.

Mathematical Activities (1)

d. Identifying quantities in their surroundings that are proportional, and solving problems using proportionality.

Chapter 3. Content by Grades

The objective of grade 6 is to enhance students' ability to think about and reason with functions as a way to summarize the viewpoints of quantitative relations taught so far, through examinations of co-varying quantities with a focus on proportional relationships.

a. Proportional relationships in mathematical expressions, tables, and graphs

Proportional relationships have the following characteristics:

(a) Suppose there are two quantities, A and B; if one of them becomes 2, 3, 4, ... times as much, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... times as much, the other changes in the same

way and becomes 2, 3, 4, ... times as much, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... times as much.

- (b) Generalizing (a), when one of two quantities becomes *m* times as much, the corresponding quantity of the other also becomes *m* times as much.
- (c) The quotient of two corresponding quantities remains constant.

By the end of grade 5, in regard to the relationship between two quantities that vary simultaneously, teaching focused on examining how quantities corresponded and changed by using tools such as tables. Particularly, in grade 5, proportional relationships in simple cases were taught.

In grade 6, in order to summarize the relationship between two quantities, a relationship with the characteristic described in (a) is defined as a *proportional relationship*. It is important to incorporate activities such as representing two quantities that vary simultaneously in everyday life using tables, etc. and identify proportional relationships by examining the characteristics of change.

For students, it is not necessarily easy to generalize (a) in the form of (b). Therefore, it is important to incorporate activities such as actually examining various numerical values from tables so that students can become aware of the viewpoint in (b).

The characteristic described in (c), focusing on the quotient of two corresponding values, is effective when examining whether two quantities are in a proportional relationship from the viewpoint of functional relationships. It is necessary to teach students so that they understand this way of examining correspondence.

A mathematical expression that represents the proportional relationship, if the quotient in (c) is *k*, is $y = k \times x$.

Generally speaking, a graph that represents a proportional relationship is a straight line passing through the origin. This is an important characteristic used to distinguish proportional relationships. Here, it is important to teach students using concrete numbers, and through the activities such as representing on a graph two quantities that vary simultaneously so that they understand that if two quantities are in a proportional relationship, the graph representing this relationship is a straight line.

b. Solving problems using proportional relationships

In grade 6, multiplication, fractions, ratio, and proportional relationships are summarized from the perspective of proportional relationships, and students will deepen their ability to think about and reason with functions by using proportional relationships to solve problems. Provide situations where proportional relationships are used effectively, and help students appreciate the value of using proportional relationships to solve problems efficiently, and foster their attitude to willingly use proportional relationships when solving everyday problems.

c. Inverse proportional relationships

The objective is to know about inverse proportional relationships and to deepen one's understanding of proportional relationships. Inverse proportional relationships have the following characteristics:

- (a) Suppose there are two quantities A and B; if one of them becomes 2, 3, 4, ... times as much, the other becomes $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... times as much, and if one of them becomes $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ... as much, the other becomes 2, 3, 4, ... times as much.
- (b) Generalizing the characteristic in (a), when one of the two quantities becomes *m* times as much, the corresponding quantity of the other becomes

 $\frac{1}{m}$ times as much.

(c) The product of the two corresponding quantities remains constant.

When teaching, it is important to compare proportional relationships and inverse proportional relationships. For inverse proportional relationship graphs, take several points that satisfy the inverse proportion relationships, connect those points, and examine the change.

Mathematical Activities (1) d

Identifying quantities in their surroundings that are in proportional relationship, and solving problems using proportionality

The objective of this activity is for students to be able to use proportional relationships, such as finding phenomena that show a proportional relationship in daily life or solving problems efficiently. Feeling and experiencing the joy of studying mathematics and appreciating its significance are also important objectives.

The following are examples of problem situations that can be solved efficiently using proportional relationships:

You have to count a lot of sheets of paper. You become aware that the number of sheets and the weight of the paper are in a proportional relationship. Therefore, if you weigh several sheets of paper and note that the total weight is \Box times as much as the weight of those sheets, then the total number of sheets of paper is also \Box times as many as the number of the sheets you weighed.

Or:

You need to find the length of a coil of wire. You think you can use the fact that the length and the weight of wire are in a proportional relationship. You take out a certain length of wire and weigh it. If you note that the total weight of the coil of wire is \triangle times as much, then the total length of the whole wire is also \triangle times as long as that of wire you weighed.

Many other situations can be found.

It is important to enable students to find these types of problems by themselves, and to foster the attitude to willingly utilize proportional relationships to deal with various situations in daily life and in the study of mathematics.

D (3) Mathematical expressions with letters

- (3) Students will deepen their understanding of mathematical expressions that represent quantitative relationships and be able to use them.
 - a. To represent numbers and quantities in mathematical expressions by using letters such as *a*, *x*, etc. instead of using words, \Box , and \triangle ; and to investigate relationships by substituting numbers for the letters.

The objective of grade 6 is to deepen the understanding of mathematical expressions that represent quantitative relations, and to be able to use mathematical expressions to represent such relationships, and to interpret them.

a. Mathematical expressions with letters such as a and x

In grade 4, students learned to use symbols such as \Box , \triangle , in mathematical expressions to represent relationships among numbers and quantities, and to investigate their relationships by substituting numbers for the symbols. In grade 6, based on what was taught in grade 4, letters such as *x* and *a* are used instead of words or symbols like

 \Box , \triangle , etc., and students are expected to gradually become accustomed to using letters.

In the lower secondary school, letters are used extensively. For a smooth transition to lower secondary school mathematics, it is important to cultivate a foundation for appreciating the merits of using literal symbols like *a* and *x*, such as conciseness.

When teaching, first make sure that students understand the use of \Box and riangle, and

then help students use *a* or *x* in place of \Box or \triangle . Consideration should be taken so that students understand, through activities such as replacing letters with numbers, that not only whole numbers but also decimal numbers and fractions can be substituted, and that they can think in terms of an expanded range of numbers.

D (4) Analyzing data

- (4) Students will investigate the distribution of data, and be able to consider and represent the data statistically.
 - a. To become aware of the mean of data.
 - b. To become aware of how tables and graphs are used to show frequency distribution.
The objective of grade 6 is to improve the ability to analyze and represent data statistically through discussion of topics such as: the mean as a representative value; frequency tables; and histograms.

a. Mean

Students learned about the mean of measured values in "B. Quantities and Measurement"-(3) in grade 5. Based on this, in grade 6 students are expected to understand the mean as a representative value of data, and deepen their understanding of mean.

When data are scattered over a range, the mean is often used to represent the entire data in order to grasp the tendency of the data. In grade 6, based on the idea of evening out data when there are several values, the *mean* is taught as a representative value of a group when there are several values.

Also, teach that distribution is another characteristic of data sets. It is necessary for students to understand that, even if the mean is the same, the characteristics of data differ if the data are concentrated or dispersed. To help students understand this, it is important to incorporate activities where students devise different ways to represent data such as by recording values on a number line.

It is important to consider enhancing students' ability to examine and represent things around them statistically using the mean.

b. Tables and graphs representing frequency distribution

Teach students that frequency tables and histograms are good methods for representing the overall distribution and characteristics of the data when they are scattered over a range, and help them to draw and interpret them.

A *frequency table* is a table in which the range of a variable is divided into several intervals (called *categories*), and each interval and its frequency is displayed, so that investigation of how the data are distributed becomes easy.

For the histogram, it is enough to understand it as comprising rectangles for each category with the horizontal dimension as the interval and the vertical dimension as the frequency.

An important idea of statistical manipulation is that data can be according to a purpose. How well data are sorted and organized—for example, how wide an interval is—greatly influences how easily the tendencies and characteristics of the data can be grasped; therefore, consideration should be taken in these areas as well.

D (5) Possible outcomes

(5) Students will be able to systematically analyze all possible outcomes for actual events.

The objectives of grade 6 are to observe all possible cases from appropriate points of view, to sort and organize them, and to list them in order, based on grade 5 activities of classification and organization.

"Systematically analyze all possible outcomes" means to organize them in order rather than randomly, which may easily cause omissions or duplications—so that all possible outcomes are clearly and correctly shown.

When teaching, an emphasis should be on systematic analysis rather than obtaining the total number of outcomes. It is necessary to take consideration to foster the attitude to systematically examine the events, avoiding omissions or duplications, by representing them using diagrams and tables.

For example, think about arranging 4 people in a line. First, think about a case where A is at the head of the line. If B lines up second, then the third is either C or D. Then think about a case where C comes second, and then think about a case where D comes second. It becomes clear from the following diagram that there are 6 cases in which A comes first. Since there are also cases where B, C, and D come first, one can conclude from the diagram that there are 24 possible arrangements.



When thinking about all possible matches among four teams, a diagram or a table such as the one below permits one to investigate all the possible combinations without omissions or duplications.



In this way, it is important for students to become able to use tables and diagrams appropriately and to think logically according to the conditions. It is also important to experience and understand that representing names concisely by using symbols is effective for systematic sorting and organizing.

Mathematical Activities

- For the contents of "A: Numbers and Calculations", "B: Quantities and Measurement", "C: Geometric Figures", and "D: Quantitative Relations", instruction should incorporate mathematical activities such as those listed below.
 - a. Investigating and explaining the ways of carrying out fraction computations using words, numbers, mathematical expressions, diagrams and number lines.
 - b. Identifying units used in their surroundings and exploring how they relate to the units they studied previously.
 - c. Identifying scaled drawings and symmetric figures in their surroundings.
 - d. Identifying quantities in their surroundings that are proportional, and solving problems using proportionality.

Terms/Symbols

Line symmetry, point symmetry, :

Chapter 4: The Construction of Teaching Plans and Remarks Concerning Content

In "III. THE CONSTRUCTION OF TEACHING PLANS AND REMARKS CONCERNING CONTENT" of the *Course of Study*, "The Construction of Teaching Plans" and "Remarks Concerning Content" are stated separately.

1. Remarks concerning the construction of teaching plans

(1) Continuous teaching and smooth transition between grades

(1) The content of each grade in Section 2 should be taught continuously in the subsequent grades, as necessary. In order to master and maintain basic skills concerning numbers, quantities, and geometric figures, opportunities for practice should be provided intentionally as the occasion demands. Furthermore, teaching through appropriate repetition may be necessary to maintain content coherence throughout the grades.

In teaching mathematics, it is necessary for students to securely acquire basic and fundamental knowledge and skills about quantities and geometric figures, and it is also equally important to become able to apply the knowledge and skills in appropriate situations. In order to achieve this, it is necessary to verify that students are acquiring the knowledge and skills taught in each grade, and if it is found to be necessary, it is important to continue teaching the contents in the next grade and beyond.

Mathematics is a subject whose content is systematically and continuously organized, and whose organization is relatively clearly defined. So new contents are often taught on the basis of previously-learned contents, like adding a new layer on previous layers.

Even in cases where content was already taught in the previous year, if it is necessary for learning new content, it should be taught as necessary and appropriate to the level of the students.

For basic abilities that affect knowledge, skills, thinking, making decisions, and expressing, it is necessary to provide opportunities to practice in order to master and maintain these skills. It is important to give the appropriate opportunities for repeated practice by monitoring students' learning progress.

Applying knowledge and skills to problem-solving and daily life can also provide good opportunities to master and maintain these skills, and it should be taken into consideration when constructing teaching plans.

In this revision, a major emphasis was placed on organizing the curriculum so that the level of complexity is gradually increased across grade level by incorporating repetitions of some topics while considering the systematic path of content between grades. In order to create a smooth connection between grades, it is necessary to teach through appropriate repetition as more advanced materials are taught gradually.

(2) Relations in teaching between domains

(2) The content in each grade should be taught in relation to other domains, A, B, C, and D.

In the Course of Study, a domain is defined for each content, such as numbers, quantities and geometric figures.

The content in grade 1 through grade 6 is defined by four domains: A. Numbers and Calculations, B. Quantities and Measurements, C. Geometric Figures, and D. Quantitative Relations. But in mathematics, the content of two or more domains are often interrelated. For example, if we take a look at teaching ways to determine area, the meaning and representation of area is in "B. Quantities and Measurements" while the geometric figures that are the subjects for determining area are in "C. Geometric Figures." Meanwhile, actual calculation of area relates to "A. Numbers and Calculations," and creating formulas and analyzing them is in "D. Quantitative Relations." Therefore, while developing teaching plans, it is necessary to consider the relationships between the domains and to incorporate content learned in other domains effectively.

(3) Teaching through mathematical activities

(3) Mathematical activities play important roles in learning mathematics, such as: securely acquiring basic and fundamental knowledge and skills; fostering the abilities of logical thinking, decision-making, and expressing; and experiencing the joy and the meaningfulness of learning mathematics. Therefore, the content of each grade should be taught through mathematical activities.

The "Objectives" of mathematics begins with the phrase "through mathematical activities," and this phrase applies to the entire statement. And it describes the fundamental idea of how teaching should proceed, which is by having students engaging in mathematical activities and teachers giving proper guidance, we can achieve the objectives listed.

Concretely speaking, this means that teaching through mathematical activities is needed for all content in each domain. At the same time, this does not deny the need for explanation given by teachers or carrying out practice sessions as necessary.

"Mathematical activities" mean various activities where students willingly engage in mathematics with purpose.

What is meant by "willingly engage in mathematics with purpose" is trying to find new properties or to create new ways of thinking or to solve concrete problems. It is necessary to teach students to have a clear purpose and to willingly engage in learning in order for them to understand the meaning of quantities and geometric figures, to enhance their ability to think, to make decisions, and to express, and for them to feel the joy and meaning of learning mathematics.

For this reason, activities where students just listen to teachers' explanations or complete practice problems are not included in mathematical activities.

Mathematical activities can include various activities. Hands-on activities, experimental activities, physical activities, and activities that use concrete objects are often considered to be typical examples of mathematical activities. But there are others.

Thinking about mathematical problems, building on mathematical knowledge and applying that knowledge, representing and explaining what students think—these do not deal with concrete objects but are included in mathematical activities.

Generally, in grades 1 and 2, the main mathematical activities are hands-on and experimental. Later grades include more activities that involve thinking and representation.

In this revision, in order to enrich mathematical activities and to clarify their role in mathematics lessons, actual examples are included in the content of each grade. The mathematical activities stated there can be implemented as they are, or similar activities can be added. Furthermore, activities not mentioned can be created and introduced by schools and teachers.

If we consider the characteristics of mathematical activities, they can be categorized in the following way:

- hands-on activities such as making things by hand
- experimental activities where students actually try out something or confirm reported phenomena inside or outside the classroom
- activities that use things in the students' environment, such as investigating situations or quantities
- exploratory activities where students can find the meaning and properties of quantities or geometric figures, or create new ways of solving problems
- activities where students can develop advanced concepts from material already learned
- activities where students apply what they have learned to various situations
- activities where students integrate what they have learned in mathematics and other subjects.

(4) Relationship to moral education

(4) Based on the aim of moral education (Chapter 1, I (2) and III (1)), the content of moral education must be taught in mathematics appropriately by considering its relationship to the teaching in moral education class.

The Course of Study, Chapter 1, I (2) states the following:

Moral education in school is carried out throughout all educational activities, with the moral education class playing a central function. Moral education should be taught appropriately, according to the developmental stages of students, with consideration to the special characteristics of each class: moral education class, general subjects, foreign language learning activities, general study, or other special activities.

Below, we state how we need to teach moral education in mathematics, considering properties of the subject of mathematics.

Moral education in mathematics should be taught appropriately, while considering learning activities, students' learning attitude, the influence of teachers' attitude and behavior, and with awareness of the relationships of the objectives of mathematics with moral education as described below.

In mathematics education, the objectives are stated as follows:

Through mathematical activities students will acquire basic and fundamental knowledge and skills about numbers, quantities, and geometric figures, cultivate their ability to consider phenomena from their daily lives with foresight to generate and organize logical thinking steps to follow through, and to represent those phenomena, to recognize the joy of mathematical activities and the value of mathematical manipulation, and to foster a disposition to willingly make use of mathematics in daily life and studies.

The ability to use foresight to generate and organize logical thinking and to represent ideas can equally be applied to the cultivation of abilities in making moral judgments. Also, attempting to apply mathematical thinking to problems in daily life and study can be considered as fostering the attitude to willingly apply one's own ideas to make life and learning better.

Next, the relationship to moral education lessons, the centerpiece in moral education, should be considered.

Sometimes it is effective to use the instructional materials used in mathematics in the moral education lessons. When applying the materials used in moral education lessons in mathematics, it should be done in a way that utilizes what was achieved in moral education class. For this reason, when making annual teaching plans, it is important to consider the content and schedule in relation to the annual plan of moral education so that the two subjects can mutually enhance one another.

2. Considerations of remarks concerning content

(1) Grasp approximate size and shape and judge adequately

(1) Teachers should help students to develop a rich sense of numbers, quantities, and geometric figures; help students to be able to grasp approximate size and shape to make appropriate decisions; and devise efficient procedures.

It is important to foster a rich sense of numbers, quantities, and geometric figures; it affects the contents in each grade. In mathematics class, it is important to teach by giving consideration to helping students grasp approximate numbers, quantities, and shapes, and make appropriate judgments based on this understanding, and helping them think of new and efficient procedures.

By grasping approximate size and shape, students can have foresight about how to progress and can prevent major errors. In problem solving, it is necessary to emphasize the clarification of purposes for which an estimate or approximate numbers are to be used, and carry out the process with foresight of approximate results and methods.

For this reason, beginning in grades 1 and 2, it is necessary to make a connection to approximation such as estimating the results of computation so that students will develop a basis for understanding approximate size and shape.

When students make estimations, they tend to use what they have already learned, exhibiting various ways of thinking and expressing, so teaching should respond to this appropriately.

(2) Activities to communicate with others by expressing ideas

(2) In order to foster students' abilities in logical thinking, reasoning, and representing, teachers should include learning activities such as: thinking and explaining using words, numbers, mathematical expressions, diagrams, tables, and graphs; and exchanging ideas with each other by representing these ideas throughout the content at each grade level.

The ability to think, reason, and express plays an important role in carrying out logical steps and in communicating intelligently. The results of the national survey indicate that students tend to have problems expressing their ideas. To foster these abilities in teaching, it is necessary to actively implement learning activities that involve thinking and explaining using words, numbers, mathematical expressions, diagrams, tables, and graphs, and that help students express and exchange their own ideas with others.

Mathematics has the special characteristic that in addition to words, students can use numbers, mathematical expressions, diagrams, tables, and graphs. It is important to

teach in a way where students can learn how to represent mathematically and apply mathematical representations.

(3) Teaching terms and symbols

(3) "Terms and Symbols" in each grade level are offered to clarify the depth and breadth of the content in a specific grade level. Terms and symbols should be taught in conjunction with the appropriate contents; students should understand the merit of using terms and symbols in their reasoning and representations.

This statement clarifies the intent of the items in "Terms and Symbols" in each grade and points of consideration for teaching them.

It is important that students use the terms and symbols. Furthermore, consideration must be given to help students understand the value of using them.

(4) Skill in calculating by using standard algorithms and estimation of results

(4) Teachers should help students to not only master calculations by using standard algorithms but also to be able to estimate the result of calculations and make adequate decisions about the methods and results of calculations. Furthermore, when teaching the contents of (A) in lower grades, teachers should help students adequately use a *soroban* [Japanese abacus] and other concrete tools to deepen students' understanding of the meaning of numbers and calculations.

It is stated that it is important to acquire skills in calculation by using standard algorithms as a foundational ability for further study of mathematics. In teaching algorithms, one should not just teach students the standard algorithms mechanically; rather, algorithms should be derived by thinking about the structure of numbers and the meaning of operations. It is also necessary that students estimate the results of calculation so that they can make appropriate judgments about the results and the procedures of calculation. Moreover, especially in lower grades, since it is necessary to deepen students' understanding of the meaning of numbers and calculation by using concrete objects, or to help them express their own ideas, it is suggested here that appropriate use of an abacus and other concrete objects can be effective.

(5) The use of computers

"(5) In order to develop a rich sense of numbers, quantities, and geometric figures, and to foster students' ability to use tables and graphs to represent ideas, teachers should consider using computers whenever appropriate or necessary."

This indicates that it is important to use tools such as computers for students to utilize their knowledge and skills, and to further develop their skills in creative ways. Computers should be used effectively, utilizing their special features—for example, data sorting, tables, graphs, animation of geometric figures, or mathematical experiments. Teachers should devise effective use of computers to enrich students' sense of quantities and geometric figures, and enhance their ability to express their own ideas.